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SOLUTION FOR AN INTEGRAL EQUATION WITH CONTINUOUS INTERVAL FUNCTIONS

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Suppose R is the set of real numbers and all integrals are of the subdivision-refinement type. Suppose each of G and H is a function from $R \times R$ to R and each of f and h is a function from R to R such that $f(a) = h(a)$, dh is of bounded variation on $[a, x]$, and $\int_a^x H^2 = \int_a^x G^2 = 0$ for $x > a$. The following two statements are equivalent:

(1) If $x > a$, then f is bounded on $[a, x]$, $\int_a^x H$ exists, $\int_a^x G$ exists, $(RL)\int_a^x (fG + fH)$ exists, and

$$f(x) = h(x) + (RL)\int_a^x (fG + fH)\ ;$$

(2) If $a \leqq p < q \leqq x$, then each of ${}_p\Pi^q (1 + H)$ and ${}_p\Pi^q (1 - G)^{-1}$ exists and neither is zero,

$$(R)\int_a^x [{}_t\Pi^x (1 + H)(1 + G)][(1 - G)^{-1}]dh$$

exists, and

$$f(x) = f(a)\, {}_a\Pi^x (1 + H)(1 - G)^{-1} + (R)\int_a^x [{}_t\Pi^x (1 + H)(1 + G)][1 - G)^{-1}]dh\ .$$

Introduction. In a recent paper [4], B. W. Helton solved the equation $f(x) = h(x) + (RL)\int_a^x (fG + fH)$ using product integration. All functions involved were required to be of bounded variation and the existence of various integrals was also required. In a subsequent paper [9], J. C. Helton was able to reduce the conditions placed on h to being a quasicontinuous function although other conditions such as requiring G and H to be of bounded variation were maintained. In still another paper [7], J. C. Helton was able to reduce the restrictions placed on G and H to that of being product bounded but he also used other restrictions not used in [4] or [9] such as requiring h to be a constant function and $G(r, s) = -G(s, r)$, a condition not unlike that of being additive. In this paper we are concerned with obtaining a solution for the equation $f(x) = h(x) + (RL)\int_a^x (fG + fH)$ without requiring either G or H to be of bounded variation or that $G(r, s) = -G(s, r)$ or that h be a constant function. Instead, our major restriction placed on G and