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HOMOTOPY CONDITIONS WHICH DETECT SIMPLE HOMOTOPY EQUIVALENCES

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Let X, Y, and K be compact polyhedra, let $p\colon Y\times K\to Y$ be the projection map, and let $f\colon X\to Y\times K$ be a homotopy equivalence which has a homotopy inverse $g\colon Y\times K\to X$ along with homotopies $fg\simeq \mathrm{id}$, $gf\simeq \mathrm{id}$ such that $p(fg\simeq \mathrm{id})$ and $pf(gf\simeq \mathrm{id})$ are small homotopies. In this paper we prove that if π_1 of each component of K is free abelian, then f must be a simple homotopy equivalence.

1. Introduction. All spaces in this paper will be locally compact, separable and metric, and a proper map is a map for which preimages of compacta are compact. The following is the main technical definition of this paper. If α is an open cover of Y, then a proper map $f\colon X\to Y$ is said to be an *α-equivalence* provided that there is a map $g\colon Y\to X$, an α-homotopy of $f\circ g\colon Y\to Y$ to the identity, and an $f^{-1}(\alpha)$-homotopy of $g\circ f\colon X\to X$ to the identity. Here $f^{-1}(\alpha)=\{f^{-1}(U)\mid U\in\alpha\}$, and a *$\beta$-homotopy* is a homotopy for which the *track* of each point lies in some element of β (see § 2).

In [14] Ferry used Q-manifolds to prove the following result:

If Y is a polyhedron, then there is an open cover α of Y so that for any polyhedron X and α-equivalence $f\colon X\to Y$, f must be a simple homotopy equivalence.

(For the definition of a *simple homotopy equivalence* (s.h.e.) for compact polyhedra we refer the reader to [24], and for noncompact polyhedra we refer to [19], where the designation *infinite* s.h.e. is used.) The above result represents the most general homotopy conditions that the author knows of which detect s.h.e.'s. It easily implies half of the Classification Theorem from Q-manifold theory [7, p. 88], which gives a homeomorphism condition which detects s.h.e.'s (see Theorem 2 below). On the other hand it follows from [16] that any cell-like map of polyhedra must be an α-equivalence, for every α. Therefore the above result implies that every cell-like map of polyhedra is a s.h.e., thus recapturing the main result of [5].

The purpose of this paper is to generalize the above result, while at the same time giving a proof which does not rely upon Q-manifold theory. In what follows K will be a compact polyhedron for which each Whitehead group $\mathrm{Wh}(K\times T^n)$ vanishes, where T^n is the n-torus ($T^0=\{\text{point}\}$). This includes, for example, all polyhedra K for which π_1 of each component of K is free abelian or