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A CHARACTERIZATION OF COVERING DIMENSION BY USE OF $\Delta_k(X)$

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Covering dimension, in the sense of Katětov, of a topological space X is characterized by use of $\Delta_k(X)$ which will be defined in the main discussion in terms of cardinalities of finite open covers of X.

1. Introduction. L. Pontrjagin and L. Schnirelmann [6] characterized dimension of a compact metrizable space X by use of the numbers $N_\rho(\varepsilon, X) = \min\{m \in N \mid$ the metric space (X, ρ) has a cover $\mathscr{U}$ such that $|\mathscr{U}| = m$ and $\operatorname{diam} U \leqq \varepsilon$ for every $U \in \mathscr{U}\}$. Their result is quite interesting in the sense that covering dimension, which is defined in terms of order (a kind of local cardinality) of a cover, is characterized in terms of global cardinality of a cover. J. Bruijning [1] generalized Pontrjagin-Schnirelmanns theorem to separable metric spaces by use of totally bounded metrics and to topological spaces by means of totally bounded pseudometrics.

In the present paper, we shall characterize covering dimension of topological spaces by use of a new function $\Delta_k(X)$, which will be defined later. It seems that $\Delta_k(X)$ can provide us with a neater characterization of dimension, perhaps because it does not involve the metric ρ in its definition while $N_\rho(\varepsilon, X)$ does.

2. Conventions. In the following discussions we frequently consider a finite collection $\mathscr{U} = \{U_1, \cdots, U_i\}$ of subsets of a space X such that $\bigcup\{U_j \mid 1 \leqq j \leqq i\} \supset A$ for a certain subset A of X, and a cover $\mathscr{V} = \{V_1, \cdots, V_i\}$ of A such that $V_j \subset U_j \cap A$ for $1 \leqq j \leqq i$. Then we may say: *$\mathscr{U}$ is shrunk to $\mathscr{V}$ on A.*

If $\mathscr{V}$ consists of open (closed) subsets of A, we *shrink $\mathscr{U}$ to the open (closed) cover $\mathscr{V}$ of A.* If $A = X$, we may drop the words "on A". If $\cap \mathscr{V} = \varnothing$, $\mathscr{V}$ is *vanishing.*

We shall denote by C^k_m the set of all m-element subsets of the set $\{1, 2, \cdots, k\}$ and by $\binom{k}{m}$ its cardinality, i.e., $\binom{k}{m} = k!/m!(k-m)!$.

By the *dimension* of a space X, $\dim X$, we will mean its Katětov dimension, i.e.,

$$\dim X = -1 \qquad \text{iff} \quad X = \varnothing\,;$$
$$\dim X \leqq n (n \geqq 0) \quad \text{iff every finite cover of } X\,,$$

consisting of functionally open sets, has a finite refinement, also consisting of functionally open sets and with order $\leqq n + 1$;