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IRREDUCIBLE LENGTHS OF TRIVECTORS OF RANK SEVEN AND EIGHT

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We determine the irreducible length of complex trivectors of rank less than or equal to eight. The irreducible length is invariant under the action induced by the general linear group on the underlying complex vector space. A classification under this action is available where representatives are explicitly given for each equivalence class and it is the lengths of these representatives which are determined.

In their paper [1], Busemann and Glassco consider the problem of determining the maximal irreducible length (called *length* from now on) $N(F, n, r)$ of r-vectors in $\Lambda^r U$ where U is an n-dimensional vector space over the field F. The length of an r-vector is the number of decomposable summands (blades in J. Schouten's book [5] and paper [6]) in a shortest possible representation of that r-vector. In [1] Busemann and Glassco state that "The values $N(C, 7, 3) = 5$, $N(C, 8, 3) = 7$, and $N(C, 9, 3) = 10$ have been claimed but questioned, see Schouten [3, p. 27] and [1]." The purpose of this paper is to show that $N(C, 8, 3) = 5$ (and not 7 as claimed) by determining the lengths of each of the representatives of the Gurevich classification in [2]. For sake of completeness the lengths of the rank 7 trivectors are also included. That $N(C, 7, 4) = 4$ (and not 5) is shown in [7].

Let U be a fixed 8-dimensional vector space over the complex numbex field and $\Lambda^3 U$ the space of trivectors considered. If $X \in \Lambda^3 U$ then $[X]$ denotes the intersection of all subspaces W of U for which $X \in \Lambda^3 W$. Then $\dim [X]$ is the rank of X. The letters a, b, c, q, r, s, p, t appearing in the Gurevich classification may be assumed to be independent vectors in U. Note that in [2] the "Λ" has been suppressed.

The equivalence classes are as follows.

I:	0
II:	$[abc]$
III:	$[aqp] + [brp]$
IV:	$[aqr] + [brp] + [cpq]$
V:	$[abc] + [pqr]$
VI:	$[aqp] + [brp] + [csp]$
VII:	$[qrs] + [aqp] + [brp] + [csp]$
VIII:	$[abc] + [qrs] + [aqp]$