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A WITT'S THEOREM FOR UNIMODULAR LATTICES

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Let K be a dyadic local field, $\mathfrak{o}$ its ring of integers, L a regular unimodular lattice over $\mathfrak{o}$. If x and y are vectors in L, we ask for necessary and sufficient conditions to map x isometrically to y. Trojan and James obtain conditions via a T-invariant when $\mathfrak{o}$ is 2-adic. Hsia uses characteristic sets and G-invariants for vectors and he solves the problem when $\mathfrak{o}$ is dyadic in general. We define here a new numerical invariant, the degree of a vector, which reflects more on the structure of L and the relationship between x, y and L. The Witt conditions will be stated in terms of this degree invariant.

1. Introduction. Let π be a prime element generating the maximal ideal of $\mathfrak{o}$ and let e be such that $2\mathfrak{o} = \pi^e\mathfrak{o}$. Let Q be a quadratic form on L, B its associated symmetric bilinear form. Then Q and B are connected by

$$Q(x + y) = Q(x) + Q(y) - B(x, y) .$$

The lattice L is unimodular simply means $B(L, L) = \mathfrak{o}$ and det L is a unit. The structure of unimodular lattices is well-known and can be found in O'Meara [7]. A vector v is primitive if $v \notin \pi L$. Hence v is primitive if and only if $B(v, L) = \mathfrak{o}$.

PROPOSITION 1. *Let v be a vector in a unimodular lattice L. Then $v \in \pi^k L$ if and only if $B(v, L) \subseteqq \pi^k\mathfrak{o}$.*

Proof. The necessity is trivial. Assume $B(v, L) \subseteqq \pi^k\mathfrak{o}$ and h is the highest power of π that divides v, that is, $v = \pi^h w$ for some primitive vector w. Hence $B(w, L) = \mathfrak{o}$ and $B(v, L) = B(\pi^h w, L) = \pi^h\mathfrak{o}$ and $h \geqq k$.

If $z \in L$ satisfies $\operatorname{ord} Q(z) \leqq \operatorname{ord} B(z, L)$, then the map σ_z given by

$$\sigma_z(v) = v - B(v, z)z/Q(z)$$

is an integral isometry known as the reflection of z. The group of integral isometries is denoted by $O(L)$. O'Meara and Pollak [8], [9] have shown that $O(L)$ is generated by reflections except in a few cases when the residue field $\mathfrak{o}/\pi\mathfrak{o}$ contains only two elements, and that in the exceptional cases one extra generator given by an