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BOUNDARY CONTINUITY OF SOME HOLOMORPHIC FUNCTIONS

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For certain bounded domains D in C^n any continuous function on $D \cup \partial_{A(D)}$, which is holomorphic on D automatically extends continuously to D^-.

For a bounded domain D in C^n let $A(D)$ be the sup normed algebra of functions continuous on D^- and holomorphic on D, and let $\partial = \partial_{A(D)}$ denote its Šilov boundary, so $\partial \subset \partial D$. Of course this inclusion can be proper, and in his thesis [0] A. Aytuna raised the question of whether every bounded continuous function on $\partial \cup D$ holomorphic on D necessarily extends continuously at all points of $\partial D \backslash \partial$. Aytuna showed this held when $n \geqq 2$ for the half-ball $D = \{z: |z| < 1, \operatorname{Re} z_1 > 0\}$, where $\partial D \backslash \partial = \{z: |z| < 1, \operatorname{Re} z_1 = 0\}$ is a union of analytic discs and normal family arguments apply. In fact there are simple domains for which continuous extension fails, as we shall see below (§ 3), while it holds rather trivially for starlike domains; our purpose here is to point out some classes of domains for which it holds, and indeed something stronger obtains, by virtue of some elementary function algebra facts combined with the Oka-Weil approximation theorem.

Recall that $K \subset D^-$ is a peak set for $A(D)$ if there is an f in $A(D)$ with $f(K) = 1$ and $|f| < 1$ on $D^- \backslash K$. $P(K)$ will denote the closure in $C(K)$ of the analytic polynomials and $H^\infty(D)$ the bounded holomorphic functions on D.

THEOREM 1. *Suppose $\partial D \backslash \partial$ is differentiable and covered by a union of peak sets K for $A(D)$, for each of which*

(1.1) *f holomorphic near K implies $f|K$ is uniformly approximable by polynomials, and*

(1.2) *$x \in K \backslash \partial$ implies $(0, \varepsilon_x)\nu_x + K \subset D$, where ν_x is the inward unit normal to ∂D at x, and $x \rightarrow \varepsilon_x$ is a positive continuous function on $\partial D \backslash \partial$.*

If h is bounded and holomorphic on D, and, for one of our peak sets K_0, has a continuous extension to $D \cup (\partial \cap K_0)$, then h extends continuously to $D \cup K_0$.

In particular if h extends continuously to $D \cup \partial$ it extends to an element of $A(D)$; in fact in this case we need not assume h bounded on D. Hypothesis (1.1) holds if each peak set K is polynomially convex by the Oka-Weil approximation theorem (cf. [3],