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ON THE STRUCTURE OF FINITELY GENERATED SPLITTING RINGS

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In this paper the structure of finitely generated splitting rings for the Goldie theory is studied. First, right nonsingular finitely generated splitting rings with essential socle which either are right finite dimensional or are right orders in a semiprimary ring are characterized. This characterization is in terms of an explicit triangular matrix structure for R. Then right nonsingular finitely generated splitting rings with zero socle are shown to be right finite dimensional if and only if they are right orders in a semiprimary ring. An explicit triangular structure is given for this class of rings as well. For certain classes of right nonsingular right finite dimensional finitely generated splitting rings with zero socle, the structure theorem can be simplified somewhat. Then right nonsingular right finite dimensional finitely generated splitting rings are characterized as a certain essential product of a ring with essential socle and one with zero socle. Right nonsingular finitely generated splitting rings which are right orders in a semiprimary ring are shown to be a direct product of a ring with essential socle and a ring with zero socle. Finally, some comments are made showing how some of these results can be applied to bounded splitting rings and splitting rings.

1. Preliminaries. In this paper all rings R are associative with identity and all R-modules are unital. Unless indicated otherwise, all modules are right modules. A left or right R-module M will be denoted by ${}_RM$ or M_R respectively. The socle of an R-module M will be denoted by $\operatorname{soc}(M)$; the socle of R will always mean the socle of R_R unless indicated otherwise.

If M is a right R-module and $X \subseteq M$, then the right annihilator of X is denoted by $r_R(X)$ or $r(X)$ if there is no ambiguity. Similarly for a left R-module M and $X \subseteq M$, $l_R(X)$ is the left annihilator of X.

A submodule K of an R-module M is an *essential submodule* of M if $K \cap L \neq 0$ for all nonzero submodules L of M. A right (left) ideal I of R is essential in R if I is essential in $R_R({}_RR)$. Let $\mathscr{S}(R)$ denote the family of all essential right ideals of R. For any right R-module M, $Z(M) = \{x \in M: r(x) \in \mathscr{S}(R)\}$ is called the *singular submodule* of M. The singular submodule of a left R-module is defined similarly. An R-module M is *singular* if $Z(M) = M$; M is *nonsingular* if $Z(M) = 0$. A ring R is right (left)-nonsingular if