PACIFIC JOTRNAL OF MATHEMATICS
Vol. 80, No. 2, 1979

A CUSHIONING-TYPE WEAK COVERING PROPERTY

S. W. DAVIS

We discuss certain covering properties which are based on Michael's notion of a cushioned collection. In particular, we discuss property L of Bacon and certain variations on property L in connection with isocompactness and the relationship between $\aleph_1$-compactness and the Lindelöf property. We then introduce property θL which is a common generalization of property L and weak $\delta\theta$-refinability, and consider uses of this property in similar connections.

0. Introduction. There are a large number of covering properties generalizing paracompactness which have been the subjects of investigation in the last thirty years. Most of these involve some sort of generalization of the notion of locally finite refinements. For example, the metacompact, meta-Lindelöf, θ-refinable, weakly θ-refinable, $\delta\theta$-refinable and weakly $\delta\theta$-refinable spaces are all classes defined by covering properties of this type.

In 1970, Bacon presented a covering property, property L, which is based on a generalization of Michael's characterization of paracompactness in terms of cushioned refinements. We present certain variations on property L and discuss how these may be used as hypotheses in place of weak $\delta\theta$-refinability in a number of theorems.

We present a new covering property, property θL, which is a common generalization of property L and weak $\delta\theta$-refinability and still implies isocompactness [every closed countably compact subset is compact]. We discuss certain variations on property θL, and find that many of the results obtained using property L or weak $\delta\theta$-refinability remain true when one of these variations is used in place of property L or weak $\delta\theta$-refinability in the hypothesis. In particular, we establish a number of results relating $\aleph_1$-compactness with the Lindelöf property and with closed completeness.

We now list certain conventions which will be used in this paper. A perfect mapping is a closed continuous function with the property that the inverse image of each point in the range is compact. We indicate a function f whose domain is the set A and whose range is contained in the set B by $f: A \to B$. For a collection $\mathscr{A}$ of sets, $\cup \mathscr{A} = \cup\{A: A \in \mathscr{A}\}$ and ord $(x, \mathscr{A}) = |\{A: x \in A \in \mathscr{A}\}|$. If $f: X \to Y$ and $A \subset X$, we write $f(A)$ to indicate the set $\{f(x): x \in A\}$.

We include for the benefit of the reader the following definitions. Original sources are listed in [16].

DEFINITION 0.1. Suppose X is a space and $\mathscr{W}$ is an open cover