PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 2, 1979

CHARACTERIZING REDUCED WITT RINGS II

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We have recently given a recursive construction of all reduced Witt rings of fields with finitely many places into the real numbers. In this paper we extend the construction to include all reduced Witt rings of fields. We then demonstrate how this recursion process can be used to prove facts about these rings.

1. Introduction. Given a field F, we denote its Witt ring of nondegenerate symmetric bilinear forms by $W(F)$. Modulo the nilradical, we obtain the reduced Witt ring $W_{\mathrm{red}}(F)$. We begin with some notation and basic facts about these rings from [11] and [12] which will be used throughout this paper. Our interest is only in fields F for which the Witt ring is not entirely torsion. This is equivalent to the field being formally real [13]. In this case, $W_{\mathrm{red}}(F)$ can be naturally embedded in $\mathscr{C}(X, \boldsymbol{Z})$, the ring of continuous functions from the topological space $X = X(F)$ of minimal prime ideals of $W(F)$ with the Zariski topology to the ring of integers with the discrete topology. One can also identify X with the set of orderings of F and with the set of ring homomorphisms from $W(F)$ to $\boldsymbol{Z}$. In dealing with elements of X, we shall usually think of them as orderings, represented by a set of positive elements P. More generally, one can consider "abstract Witt rings" (defined in [12]) and obtain a similar embedding. We will generally identify such rings with their canonical embeddings in rings of continuous functions, hence considering an element f in $W_{\mathrm{red}}(F)$ as a continuous function $f: X(F) \to \boldsymbol{Z}$.

Our goal in this paper is to separate the reduced Witt rings of fields from among the class of all abstract Witt rings. This will be done by extending results in [6] to obtain a ring theoretic construction of a category of rings whose objects represent all isomorphism classes of reduced Witt rings of fields. The recursive nature of this construction provides a strong method of proof for questions concerning the ring structure of the reduced Witt ring. This will be demonstrated in §4 where we obtain a new proof of a powerful recent theorem due to Becker and Bröcker [1, Theorem 5.3]. In §3 we state and prove the characterization theorem for reduced Witt rings, generalizing [6, Theorem 2.1].

Section 2 is devoted to defining and briefly studying the category of rings in which we are interested. As a matter of notation, we shall let R^* denote the multiplicative group of units in any ring R.