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ISOMORPHISMS AND SIMULTANEOUS EXTENSIONS IN $C(S)$

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Let h map a subspace A continuously into the completely regular space S so that A and $h[A]$ are completely separated in S, and let Q be the quotient space of S gotten by identifying p with $h(p)$ for all p in A. If there exists a simultaneous extension from $C(A)$ into $C(S)$, then there exists an isomorphism of $C(S)$ onto itself, taking $C(Q)$ onto $C(S \| A)$, which is the identity on $C(S \| h[A])$ (whence $C(Q)$ is complemented in $C(S)$). The converse holds providing A and $h[A]$ are normally embedded in S and h is a homeomorphism.

Introduction. Our main results, stated above, are Theorems 2.3 and 2.5 of §2. In §1 we establish these results in the special case when S is the topological sum of two disjoint spaces S_1 and S_2, where $A \subset S_1$ and $h[A] \subset S_2$. We also show, in Theorem 1.3, that isomorphisms of the above type always exist whenever h maps all of S_1 into S_2.

In §2 the results of §1 are extended to the general case by means of Lemma 2.1, which enables us to recover S in a natural way as a quotient space of a topological sum of two disjoint spaces.

Background. $C(S)$ denotes the Banach space of all bounded continuous real or complex valued functions on S with supremum norm.

Let A be a subspace of S. A is *normally embedded* in S if every element of $C(A)$ has a continuous extension to S. $C(S \| A)$ (resp. $C(S, A)$) denotes the subspace of all functions in $C(S)$ that are zero (resp. constant) on A and $\Lambda(A, S)$ denotes the set of all *simultaneous extensions* (bounded linear operators that extend functions) from $C(A)$ into $C(S)$. For further information on simultaneous extensions, see [2], [3] and the references therein.

Two Banach spaces X and Y are *isomorphic* (write $X \sim Y$) if there exists a one-to-one bicontinuous linear operator from X onto Y. A subspace Z of X is *complemented* in X if there exists a projection of X onto Z in the sense of [6, p. 480].

βN denotes the Stone-Čech compactification of the integers.

1. Throughout this section $S_1 \oplus S_2$ denotes the topological sum (free union) of the disjoint completely regular spaces S_1 and S_2, A a subspace of S_1, h a continuous function from A into S_2, and Q the