PACIFIC JOURNAL OF MATHEMATICS
Vol. 80, No. 2, 1979

ON THE MULTIPLICATIVE COUSIN PROBLEMS FOR $N^p(D)$

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Let D be a strictly convex domain in C^n with C^2-class boundary. Let $N^p(D), 1 < p < \infty$, be the set of all holomorphic functions f in D such that $(\log^+|f|)^p$ has a harmonic majorant. The purpose of this paper is to show that the multiplicative Cousin problems for $N^p(D), 1 < p < \infty$, are solvable.

1. Introduction. Let D be a domain in C^n. We denote by S_n the class of bounded domains D in C^n with the properties that there exists a real function ρ of class C^2 defined on a neighborhood W of ∂D such that $d\rho \neq 0$ on $\partial D, D \cap W = \{z \in W: \rho(z) < 1\}$ and the real Hessian of ρ is positive definite on W. For $1 \leqq p \leqq \infty$, we denote by $N^p(D)$ the set of all holomorphic functions f in D such that $(\log^+|f|)^p$ has a harmonic majorant in D. When $p = \infty$, we assume that $|f|$ is bounded in D. When $p = 1, N^1(D)$ is the Nevanlinna class. E. L. Stout [5] proved that the multiplicative Cousin problem with bounded data on every domain of class S_n can be solved. In this paper we shall prove that the multiplicative Cousin problems for $N^p(D), 1<p\leqq\infty$, can be solved. The proof depends on the Riesz type theorem concerning conjugate functions and the estimates obtained by E. L. Stout [5], [6]. The required analysis is available on strictly pseudoconvex domains, but the geometric patching constructions in §3 depend on euclidean convexity. Explicitly, the above results are the following:

THEOREM. *Let $D \in S_n$. Let $\{V_\alpha\}_{\alpha \in I}$ be an open covering of $\bar{D}$, and for each $\alpha, f_\alpha \in N^p(V_\alpha \cap D), 1 < p \leqq \infty$. If for all $\alpha, \beta \in I, f_\alpha f_\beta^{-1}$ is an invertible element of $N^p(V_\alpha \cap V_\beta \cap D)$, then there exists a function $F \in N^p(D)$ such that for all $\alpha \in I, Ff_\alpha^{-1}$ is an invertible element of $N^p(V_\alpha \cap D)$.*

In the case when D is an open unit polydisc in C^n, theorem for $p = 1$ was proved by S. E. Zarantonello [7], and theorem for $p = \infty$ was proved by E. L. Stout [4].

Let $A(D)$ be the sheaf of germs of continuous function on $\bar{D}$ that are holomorphic in D. I. Lieb [2] proved that $H^q(\bar{D}, A(D)) = 0$ for $q > 0$, provided D is a strictly pseudoconvex domain with C^5-boundary. Let $D \in S_n$ and let D have a C^5-boundary. Then, from the above Lieb's result and $H^2(D, Z)=0$, by applying the standard exact sequence of sheaves