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M-IDEALS IN $B(l_p)$

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This paper is concerned with the M-ideal structure of the algebras $B(l_p)$ of bounded operators on the sequence spaces l_p, $1 < p < \infty$. The M-summands are completely determined, but the M-ideals are only partially characterized. However evidence is presented to support the conjecture that the only nontrivial M-ideal is the ideal $C(l_p)$ of compact operators on l_p.

1. Introduction. It has been observed by several authors that various structure theorems for $B(H)$, H a separable Hilbert space, can be extended to the spaces $B(l_p)$, $1 < p < \infty$. For instance, it is known that the ideal $C(l_p)$ of compact operators in $B(l_p)$, $1 < p < \infty$ is the only closed nontrivial two sided ideal [9], and Các [5] has shown that the second dual of this space is isometrically isomorphic with $B(l_p)$. In another direction Hennefeld [10] proved that $C(l_p)$ is an M-ideal in $B(l_p)$, $1 < p < \infty$. The notion of an M-ideal generalizes the two sided ideals in a C^*-algebra; due to the geometric characterization of the ideals in these special algebras the M-ideals have been identified with the two sided ideals [13].

The present work arose from an attempt to extend the latter result on M-ideals to $B(l_p)$, $1 < p < \infty$. Although the M-ideals in $B(l_p)$ are not yet completely characterized, certain positive results are obtained. For instance, in $B(l_p)$ the M-summands, a special subclass of M-ideals, are described. Moreover, it is shown that $C(l_p)$ is a minimal M-ideal in $B(l_p)$, $1 < p < \infty$, in the sense that every nontrivial M-ideal in $B(l_p)$ contains the ideal of compact operators. The techniques developed herein yield a new proof that the M-ideals must be two sided ideals in a C^*-algebra. In addition, certain structure theorems on the state space of $B(l_p)$, $1 < p < \infty$ and on the hermitian elements of $B(l_p)^{**}$ are derived.

2. Preliminaries. A closed subspace N of a Banach space X is said to be an L-ideal if there exists a closed subspace N' such that $X = N \oplus N'$ and $\|n + n'\| = \|n\| + \|n'\|$ for all $n \in N$ and $n' \in N'$. A closed subspace J is said to be an M-ideal if the annihilator $J^\perp$ is an L-ideal in X^*. A closely related concept is that of an M-summand which is defined to be an M-ideal J with a complementary closed subspace J' such that $\|j + j'\| = \max\{\|j\|, \|j'\|\}$ for all $j \in J$ and $j' \in J'$. It should be noted that M-ideals need not be M-summands. The detailed properties of these objects have been studied in [2],