PACIFIC JOURNAL OF MATHFMATICS
Vol. 81, No. 2, 1979

OPERATORS OVER REGULAR MAPS

STEPHEN E. WILSON

In this paper, we define certain operators, each of which transforms one regular map into another. These operator are based on the notions of Petrie path and jth order "hole" introduced by Coxeter. Together with the usual dual operator, they are a powerful tool for the analysis and taxonomy of regular maps. We produce, as an example, 18 distinct maps from the icosahedron, including six of Brahana and Coble's eight pentagonal dodecahedra.

DEFINITIONS. A *map* is a division of a compact 2-manifold into simply connected regions called the *faces* of the map by an embedded graph or multigraph. A *flag* in a map M is a mutual incidence of a face, an edge and a vertex. A *symmetry* or *automorphism* of M is a permutation of its parts which preserves kind and incidence. The map M is to be called *regular* provided that its group of symmetries, $G(M)$, acts transitively on its flags. Consider Fig. 1: a regular map M must possess a symmetry α which interchanges the flags (A 1 V) and (A 1 U), another, β, which interchanges (A 1 V) and (B 1 V), and a third, X, which interchanges (A 1 V) and (A 2 V). These three symmetries generate $G(M)$, and we may think of them as reflections about the appropriate axes. The map also has rotational symmetries: $R = \alpha X$ (meaning first apply α and then X) is rotation one step counterclockwise about face A. $S = \beta X$ is rotation one step clockwise about V, and $\gamma = \alpha\beta$ is rotation 180° around edge 1.

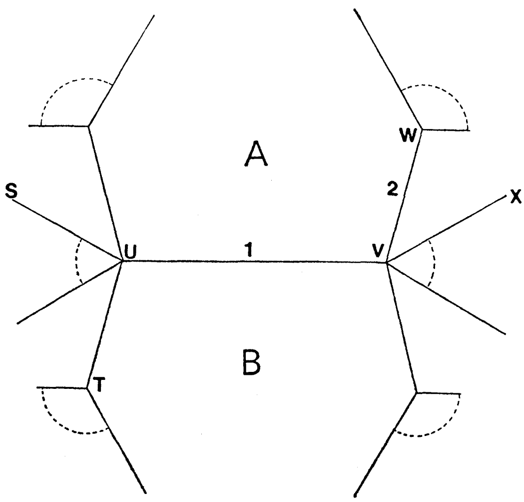

FIGURE 1. Flags in a regular map