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THE CARRIER SPACE OF A REFLEXIVE OPERATOR ALGEBRA

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Many properties of nest algebras are actually valid for reflexive operator algebras with a commutative subspace lattice. In this paper we collect a number of such results related to the carrier space of the algebra. Included among these results are a generalization of Ringrose's criterion, a description of the partial correspondence between lattice homomorphisms of the carrier space and projections in the lattice, the construction of isometric representations of certain quotient algebras, and a direct sum decomposition of the commutant of the core modulo the intersection of the spectral ideals.

Let $\mathscr{A} = \operatorname{Alg}\mathscr{L}$, where $\mathscr{L}$ is a commutative subspace lattice and let $\mathscr{I}$ be the intersection of all the spectral ideals in $\mathscr{A}$. (See §1 for definitions.) In §1 we generalize Ringrose's criterion to the commutative subspace lattice case: $A \in \mathscr{I}$ if, and only if, for each $\varepsilon > 0$ there is a finite family $\{E_i\}$ of mutually orthogonal intervals from $\mathscr{L}$ such that $\sum E_i = 1$ and $\|E_iAE_i\| < \varepsilon$, $i = 1, \cdots, n$. We also prove that $\mathscr{I}$ is the closed linear span of commutators of the form $AL - LA$, where $A \in \mathscr{A}$ and $L \in \mathscr{L}$. In §2 we describe the partial correspondence between certain projections in $\mathscr{L}$ and certain lattice homomorphisms in the carrier space X. A necessary (but not sufficient) condition for an operator A to be in the radical of $\mathscr{A}$ is given in §3. In §4 we exhibit isometric representations as algebras of operators acting on Hilbert space of each quotient algebra $\mathscr{A}/\mathscr{A}_\phi$ and of the quotient $\mathscr{A}/\mathscr{I}$. In the nest algebra case this was done by Lance in [5]. Finally, in §5 we generalize somewhat a theorem from [6] which identifies the $\mathscr{I}$-commutant of the core of $\mathscr{A}$ as the direct sum of the diagonal of $\mathscr{A}$ and $\mathscr{I}$.

1. Let $\mathscr{L}$ be a commutative subspace lattice acting on a separable Hilbert space $\mathscr{H}$, that is to say, $\mathscr{L}$ is a lattice of commuting, orthogonal projections on $\mathscr{H}$ which contains 0 and 1 and is closed in the strong operator topology. Let $\mathscr{A} = \operatorname{Alg}\mathscr{L}$, the algebra of all operators leaving invariant each projection in $\mathscr{L}$. Then $\mathscr{A}$ is a reflexive operator algebra whose lattice of invariant subspaces is just $\mathscr{L}$ [1]. Define the *carrier space*, X, of $\mathscr{L}$ to be the set of all lattice homomorphisms of $\mathscr{L}$ onto the trivial lattice $\{0, 1\}$. If the carrier space is given the topology in which a net, ϕ_ν, converges to ϕ if, and only if, $\phi_\nu(L) \to \phi(L)$ for each $L \in \mathscr{L}$, then it becomes a