

POLYNOMIAL COVERING SPACES AND HOMOMORPHISMS INTO THE BRAID GROUPS

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A polynomial covering space is the zero set for a parametrized family of complex polynomials. Such covering spaces arise naturally in several contexts in the study of holomorphic mappings. There are fruitful connections between polynomial covering spaces and certain homomorphisms into the Artin braid groups. We shall establish these connections and use them to obtain an algebraic classification of the polynomial covering spaces.

1. Introduction. Covering spaces defined by parametrized families of complex polynomials arise naturally in several contexts in the study of holomorphic mappings, e.g., in connection with the Weierstrass preparation theorem. A study of the topology of such covering spaces within a proper topological framework was suggested in [6] by the introduction of polynomial covering spaces. Roughly speaking, an n -fold polynomial covering space over a connected, topological space X is the zero set for a continuously varying family of simple, normed, complex polynomials of degree n parametrized by X . It is well known that an n -fold covering space over X is classified by a conjugacy class of homomorphisms of the fundamental group of X into the symmetric group on n letters. The purpose of this paper is to examine the relationship between n -fold polynomial covering spaces over X and homomorphisms of the fundamental group of X into the Artin braid group on n strings. What we are looking for are necessary and sufficient conditions, for a finite covering space over X to be equivalent to a polynomial covering space over X , and for two polynomial covering spaces over X to be equivalent as coverings.

In [4] and later papers, see e.g., the references cited in [5], Gorin and Lin also study parametrized families of simple, normed complex polynomials, but not from the point of view of examining the topological types of the associated covering spaces. The purpose of Gorin and Lin is to investigate, when a continuous family of polynomials admits a continuous factorization in polynomials of degree 1 over the parameter space. The existence of such a factorization is equivalent to the associated polynomial covering space being trivial. Of other papers somehow related to the work in this paper, we note in particular the papers by Arnol'd [1] and Zjuzin and Lin [9].