PACIFIC JOURNAL OF MATHEMATICS
Vol. 82, No. 1, 1979

EMBEDDING OPEN 3-MANIFOLDS IN COMPACT 3-MANIFOLDS

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We consider open 3-manifolds that are monotone unions of compact 3-manifolds each bounded by a torus. We give necessary and sufficient conditions for embedding such an open 3-manifold in a compact 3-manifold. We also show that if the open 3-manifold embeds in a compact 3-manifold, then it embeds in a compact 3-manifold as the complement of the intersection of a decreasing sequence of solid tori.

1. Introduction and preliminary definitions. Kister and McMillan [5] showed that a particular contractible open 3-manifold does not embed in S^3. Haken [4] then showed that this same open 3-manifold does not embed in any compact 3-manifold. Haken's major tool was his finiteness theorem stating that there is an upper bound on the number of incompressible nonparallel surfaces in a compact 3-manifold. See [4] and § VI of [12]. This theorem is also the major tool in this paper.

The spaces and maps that we will consider are intended to be in the piecewise linear category. An *n-manifold* is a separable metric space such that each point has a neighborhood homeomorphic to an n-cell. A *submanifold* of an n-manifold is a subspace that is also an n-manifold. A k-manifold F is *properly embedded* in an n-manifold N if and only if F is a closed subset of N and $F \cap \partial N = \partial F$. A *surface* is a connected 2-manifold; a *planar surface* is a surface that can be embedded in a disk. A *punctured disk* is a planar surface obtained by removing from the interior of a disk the interiors of a finite collection of disjoint subdisks. Similarly a *punctured 3-cell* is a compact 3-manifold obtained by removing from the interior of a 3-cell the interiors of a finite collection of disjoint 3-cells.

Let I denote the unit interval $[0, 1]$; let I_i denote the interval $[-i, i]$.

Two properly embedded surfaces F and F' are *parallel* in a 3-manifold N if and only if there is an embedding of $(F \times I, \partial F \times I)$ into $(N, \partial N)$ such that F is the image of $F \times \{0\}$ and F' is the image of $F \times \{1\}$. A collection of properly embedded surfaces is *parallel* if and only if any two disjoint surfaces in the collection are parallel. The corresponding definitions for parallel simple closed curves in a 2-manifold are similar.

A 2-manifold F properly embedded in a 3-manifold N is *compressible* if and only if either F is a 2-sphere that bounds a 3-cell in