PACIFIC JOURNAL OF MATHEMATICS
Vol. 82, No. 1, 1979

KO-EQUIVALENCES AND EXISTENCE OF NONSINGULAR BILINEAR MAPS

KEE YUEN LAM

We show how to use homotopy theoretic methods to construct maps between various truncated projective spaces that induce isomorphisms in KO cohomology theory. We then use these maps to establish the existence of new families of nonsingular bilinear maps.

1. Introduction. Bilinear maps $f: R^a \times R^b \to R^c$ with the *nonsingular property* that $f(x, y) = 0$ implies $x = 0$ or $y = 0$ have been of interest for several reasons: (1) they generalize the multiplication map of the classical division algebras over R; (2) they provide estimates for the geometric dimension of vector bundles over real projective spaces [10], and are hence instrumental in the study of immersions of such spaces into R^n; (3) those maps with the additional property that $|f(x, y)| = |x|\cdot|y|$ can sometimes be used to produce essential harmonic maps between Euclidean spheres, in the sense of Eells and Sampson [5]. See [14] and [9, Theorem 4.2].

Furthermore, it has been realized for some time that there is an interesting relationship between nonsingular bilinear maps and stable homotopy theory [8], [9]. The purpose of this article is to further explore that relationship. Whereas [9] deals with *known* bilinear maps and their implications in homotopy theory, the present paper describes, in § 5, how homotopy theory could in turn be used to establish the existence of *new* families of nonsingular bilinear maps. These new families are distinct from the classical examples of Hurwitz and Radon [7], and yet exhibit the same "Clifford periodicity" phenomenon which is characteristic of the Hurwitz-Radon family.

Since the Hurwitz-Radon family can be used to produce essential harmonic maps between Euclidean spheres, it would be interesting to ask whether there exist families of nonsingular bilinear maps, occurring in the same dimension ranges as the ones established in this paper, that will yield further examples of essential harmonic maps.

The main homotopy tools employed in the paper are certain maps between truncated projective spaces called KO-equivalences. Roughly, a KO-equivalence is a map inducing an isomorphism in KO-cohomology. The methods in §§ 3, 4 for constructing KO-equivalences follow closely the techniques of [6, § 4]. Such constructions might have some independent interest, and could perhaps be read on their own right.