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METABELIAN REPRESENTATIONS OF KNOT GROUPS

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The question of determining which finite metabelian groups may be the homomorphic image of a given knot group G is considered in this paper. As a starting point, it is shown that a homomorphism of a knot group onto a metabelian group H such that $[H: H']=n$ must factor through $Z_n \otimes A_n$, where A_n is the homology group of the n-fold cyclic covering space.

This is similar to a theorem of Burde [1 Satz 4], and Reyner [5] has also proven a similar result, showing in effect that such a homomorphism must factor through $Z \oslash A_n$. Now, A_n can be given the structure of a module over the ring $Z\langle t\rangle$ of L-polynomials, and the problem of determining the metabelian factor groups of G can be reduced to determining the factor modules of A_n.

In this paper, then, a necessary and sufficient condition is given (in terms of the Alexander matrix) for a knot group to have a representation onto any given metabelian group H such that H' contains no cyclic subgroup of order n^2 for any n. (See Theorem 1.5 and the remarks previous to Theorem 1.3.)

Such metabelian groups have a simple structure which is not shared by arbitrary metabelian groups. We therefore limit the scope of this paper to groups with this property. From Theorem 1.5 we deduce necessary and sufficient conditions for a knot group to have a representation on groups of more restricted classes in terms of the Alexander polynomial (Theorems 1.7 and 1.11).

The results obtained are similar in spirit to previous results of Fox [2] about metacyclic representations, and Riley [6] about A_4 representations, and in fact, their theorems are shown to be special cases of the theorems of this paper (see Examples 1.12 and 1.13).

In a final section it is shown how a table of the homology groups of the cyclic coverings may be used to determine the possible metabelian representations. This is included because of the ease with which one can deduce quite complete information from such a table, which may be calculated by computer. The Alexander matrix, of course, contains more information (in fact in a sense complete information) about metabelian representations, but the information therein is not so readily accessible.

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