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CONCORDANCE AND HOMOTOPY, I: FUNDAMENTAL GROUP

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We study the effect of a concordance on the fundamental group of the manifolds involved.

DEFINITION (A). Two submanifolds X, Y of M^n are said to be concordant if there is an embedding $c: X \times I \to M \times I (I = [0, 1])$ which is transversal on $M \times \partial I$ and $c^{-1}(M^n \times \partial I) = X \times \partial I$, $c(X \times 0) = X = 0$, $c(X \times 1) \approx Y \times 1$.

In [11], a similar definition—that of I-equivalence—is given for subcomplexes X, Y of a complex M by simply dropping all smoothness hypotheses from definition (A) and replacing them with cellularity hypotheses.

Let now G_1 be a group and G_i its lower central series (cf. § 1). Define $G_\infty = \cap G_i$ and $G = G_1/G_\infty$ ("group G_1 made residually nilpotent"). Observe $\{G_1/G_i, p_i\}$ is an inverse system where $p_i: G_1/G_{i+1} \to G_1/G_i$ is the obvious map. Let $\tilde{G}$ be its limit (nilpotent completion) which is, in general, uncountable. There is a natural inclusion $G \to \tilde{G}$. In particular, if S is a space, define $\pi(S) = \pi_1(S)/[\pi_1(S)]_\infty$ and $\tilde{\pi}(S) = [\pi_1(S)]^{\sim}$.

DEFINITION (B). Two (finitely generated) groups are *I-equivalent* if their nilpotent completions are isomorphic.

Let now X, Y be subcomplexes of M. If we have some sort of Alexander duality (v. gr. M a manifold), so that we can prove $H_q(M - X) \approx H_q(M - Y)$, then [11], *If X and Y are I-equivalent so are $\pi(M - X)$ and $\pi(M - Y)$.* The moral here is that we might as well work with residually nilpotent groups. *This we shall assume hereafter* so that we have no need of writing "G_1" for a group G. We have in mind extending the above results to concordances: let be the free group in letters $x_1, \cdots, x_r$. Define $G(x_1, \cdots, x_r)$ (or $G(x)$) as the free product $G*\Phi$. Let $\partial_i: G(x) \to Z$ be the map defined by $\partial_i | G = 0$, $\partial_i(x_j) = \delta_{ij}$. Let now $W = \{w_1, \cdots, w_r\}$ be an r-element subset of $G(x)$, and let NW be the smallest normal subgroup of $G(x)$ containing W. Assume the integral matrix $||\partial_i w_j||$ satisfies

$$(1) \qquad \det ||\partial_i w_j|| = \pm 1 .$$

Define $G(\xi_1, \cdots, \xi_r)_1$ (or $G(\xi)_1$) as the quotient $G(x)/NW$. Let $G(\xi) = G(\xi)_1/G(\xi)_\infty$, a residually nilpotent group. If $i: G \to G(\xi)$ is the map $G \to G(x) \to G(x)/NW \to G(\xi)_1/G(\xi)_\infty$, we prove i is monic and