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RIGHT SELF-INJECTIVE RINGS WHOSE ESSENTIAL RIGHT IDEALS ARE TWO-SIDED

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A ring R of the kind described by the title is called a right q-ring and is characterized by the property that each of its right ideals is quasi-injective as a right R-module. The principal results of this paper are Theorem 6, which describes how an arbitrary right q-ring is constructed from division rings, local rings, and right q-rings with no primitive idempotent, and Theorem 5 which shows that a right q-ring cannot have an infinite set of orthogonal noncentral idempotents.

Ivanov described the structure of indecomposable, nonlocal right q-rings and conjectured that every right q-ring must be a direct sum of such rings together with a ring all of whose idempotents are central. Our results imply that though the structure of right q-rings is slightly more complicated than this (there are chain q-rings), one can still reduce the study of q-rings to ones which have only central idempotents. More precisely, the study of right q-rings is reduced to the study of right self-injective duo rings which are either local or have no primitive idempotent.

The work done here is an extension and generalization of Ivanov's investigations. We develop the finiteness conditions inherent in that work without the assumption of indecomposability and the structure of an arbitrary right q-ring is developed at the same time. Throughout the paper all rings have identity $1 \neq 0$ and all modules are unital.

Preliminaries. If one has a decomposition $A = A_1 \oplus A_2 \oplus \cdots \oplus A_n$ of a right R-module A as a finite direct sum of submodules then one has a representation of $\mathrm{End}_R A$, the ring of R-endomorphisms of A, as a ring of $n \times n$ "matrices" of the form (α_{ij}) where α_{ij} belongs to $\mathrm{Hom}_R(A_j, A_i)$. In particular, when one has a finite decomposition of the module R_R one also has a representation of the ring $R \cong \mathrm{End}_R R$ as a ring of matrices. A decomposition of $R_R = A \oplus B$ as a direct sum of two modules A and B which are unrelated in the sense that $\mathrm{Hom}_R(A, B)$ and $\mathrm{Hom}_R(B, A)$ are both the trivial group yields a representation of R as the product of the rings $\mathrm{End}_R A$ and $\mathrm{End}_R B$. For a direct sum decomposition of R_R, such unrelated summands may be achieved by summing over classes of related summands. When a module M is a direct sum of simple