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# TOPOLOGICAL MEASURE THEORY FOR COMPLETELY REGULAR SPACES AND THEIR PROJECTIVE COVERS

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**This paper investigates the relationships among tight, $\tau$-additive, and $\sigma$-additive Baire measures on a completely regular Hausdorff space $X$ and its projective cover $E(X)$. The most interesting questions arise in the $\sigma$-additive case, and lead to the following definitions: the space $X$ has the weak (resp. strong) lifting property if for each $\sigma$-additive measure on $X$, some (resp., every) pre-image measure on $E(X)$ is $\sigma$-additive. It is shown that every weak *cb* space has the strong lifting property, while the Dieudonné plank fails even the weak lifting property. Also, if $X$ is weak *cb*, then $X$ is measure-compact if and only if $E(X)$ is measure-compact.**

**Some applications to extensions of measures on lattices and to strict topologies on spaces of continuous functions are given. A relationship between the lifting properties mentioned above and conventional use of the term "lifting" in measure theory is indicated.**

A topological space is said to be extremally disconnected if the closure of every open set is again open. Such a property seems remote from the topological settings usually encountered in analysis; for example, a metric space with this property must be discrete. Nonetheless, the property of extremal disconnectedness occurs with surprising frequency in many basic results of modern analysis. Here are some of them:

(1) The lattice $C(X)$ of continuous real-valued functions on a completely regular space $X$ is Dedekind complete if and only if $X$ is extremally disconnected.

(2) A Boolean algebra is complete if and only if its Stone space is extremally disconnected.

(3) If $X$ is a compact Hausdorff space, then $C(X)$ with the supremum norm is isometrically isomorphic to a dual Banach space if and only if $X$ is hyperstonian (i.e., extremally disconnected, and the union of the supports of the normal measures on $X$ is dense in $X$).

This paper is concerned with Baire measures on completely regular spaces. The critical fact which motivates the work is that for each completely regular Hausdorff space $X$, there is an extremally disconnected space $E(X)$, called the projective cover or absolute of $X$, and a perfect irreducible map $\kappa$ of $E(X)$ onto $X$. We can