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CONVOLUTION CUT-DOWN IN SOME RADICAL CONVOLUTION ALGEBRAS

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Let $\mathscr{A}=L^1_{\text{loc}}(\boldsymbol{R}^+)$ be the algebra of locally integrable functions on the positive real axis, with convolution as multiplication, given by

$$(f*g)(x)=\int_0^x f(x-t)g(t)dt \, .$$

Sometimes it is convenient to think of our functions as being defined on all of $\boldsymbol{R}$, but vanishing for negative x. We are interested in subalgebras of $\mathscr{A}$ that are Banach algebras in some norm, and that are radical in the sense that there exist no (nontrivial) complex homomorphisms. We call these algebras *radical convolution algebras*. Such algebras A present a challenge because there is no Fourier transform for them.

We are concerned with the problem of "convolution cut-down"; namely whether given a radical convolution algebra A and an $f \in A$, there must exist an $h \in A (h \neq 0$ such that $f*h \in L^1(\boldsymbol{R}^+)$. We show, at least, that one cannot always choose $h \in L^1$. As a corollary, we show that *simultaneous* convolution cut-down is not always possible.

A class of examples is formed by certain Beurling algebras $A_w = L^1_w(\boldsymbol{R}^+)$, where w is a positive weight function that satisfies

$$\text{(i)} \qquad w(x+y) \leqq w(x)w(y)$$

and where

$$\text{(ii)} \qquad ||f|| = \int_0^\infty |f(t)|\, w(t)dt \, .$$

It is not hard to show that for A_w to be a radical algebra, it is necessary and sufficient that

$$\text{(iii)} \qquad [w(x)]^{1/x} \longrightarrow 0 \quad \text{as} \quad x \to +\infty \, .$$

For example, we could choose $w(x) = \exp(-x^\alpha)$ for any $\alpha > 1$. (Note that the restriction, assumed by many authors, that $w(x) \geqq 1$, certainly does not apply here.)

Problem. Given a radical convolution algebra A and an f in A, does there exist an h in A ($h \neq 0$) such that $f*h \in L^1$?

This problem arose in discussion with Jamil A. Siddiqi, whom the author thanks for his help. An affirmative answer to the pro-