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WEAK FROBENIUS RECIPROCITY AND COMPACTNESS CONDITIONS IN TOPOLOGICAL GROUPS

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We study weak containment relations between unitary representations of a locally compact group G and closed subgroups H. We prove that certain weak Frobenius properties and compactness conditions are equivalent. Moreover, for amenable G having small invariant neighborhoods at e weak Frobenius reciprocity (FP) defined by Fell holds for the pair (G, H) if every element of H has relatively compact conjugacy class in G.

Introduction. In [4], Fell considers the following weak version of the Frobenius reciprocity property (FP): for every closed subgroup H of a locally compact group G and $\pi \in \hat{G}$, $\psi \in \hat{H}$ *π is weakly contained in ${}_GU^\psi$, the unitary representation of G induced by ψ, if and only if ψ is weakly contained in the restriction $\pi|H$ of π to H.*

Compact groups have property FP by the classical reciprocity theorem; Fell has shown that abelian groups satisfy FP.

In §2 we deal with a weaker property (RFP): reciprocity above holds for every $\psi \in \hat{H}$ and the trivial one dimensional representation I_G of G (not necessarily for arbitrary $\pi \in \hat{G}$). Property RFP is inherited by closed subgroups, we do not know whether this is true for FP. However, we have shown in [8] that for discrete groups G properties FP and RFP are equivalent with G to have only finite conjugacy classes. To get analogous results in the nondiscrete case we look at the normal subgroup G_F of G, the union of all relatively compact conjugacy classes in G. G_F is open if and only if there is a compact neighborhood of $e \in G$, invariant under the action of G by inner automorphisms ($G \in$ [IN]; see [15], for a proof). It turns out for the class of IN-groups RFP to be a compactness condition.

THEOREM A. *For a locally compact group the following conditions are equivalent*

(1) $G \in [IN] \cap [RFP]$

(2) $G = G_F$.

Also for Lie groups $G \in$ [RFP] G_F is open as it will be shown in [3]. Thus it follows from Theorem A, that for Lie groups or connected groups $G \in$ RFP is equivalent with G to have only relatively compact conjugacy classes ($G \in [FC]^-$).

If G is an IN-group there is a compact normal subgroup K of