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A PRIMENESS PROPERTY FOR CENTRAL POLYNOMIALS

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In this note we prove an anolog of of Amitsur's theorem for central polynomials.

THEOREM. Let F be an infinite field, $f(x) = f(x_1, \cdots, x_r)$, $g(x) = g(x_{r+1}, \cdots, x_s)$ two noncommutative polynomials in disjoint sets of variables. Assume that $f(x_1, \cdots, x_r) \cdot g(x_{r+1}, \cdots, x_s)$ is central but not an identity for F_k. Then both $f(x)$ and $g(x)$ are central polynomials for F_k.

Note. Since $[x, y]^2$ is central for F_2 while $[x, y]$ is not, the assumption of disjointness of the variables cannot be removed.

Central polynomials that are not identities of the $k \times k$ matrices F_k were constructed in [2], [3]. In [1] Amitsur proved the following primeness property of the polynomial identities of F_k:

THEOREM (Amitsur). *Let F be an infinite field, $f(x) = f(x_1, \cdots, x_n)$, $g(x) = g(x_1, \cdots, x_n)$ two noncommutative polynomials over F. If $f(x) \cdot g(x)$ is an identity for F_k, then either $f(x)$ or $g(x)$ is an identity for F_k.*

Proof of the theorem. Since F is infinite, by standard arguments we may assume it is algebraically closed. Hence every matrix in F_k is conjugate to its Jordan canonical form. We show (W.L.O.G.) that $f(x)$ is central. By assumption there are $y_1, \cdots, y_s \in F_k$ such that

$$f(y_1, \cdots, y_r) \cdot g(y_{r+1}, \cdots, y_s) = \alpha I \neq 0 .$$

Denote $A = g(y_{r+1}, \cdots, y_s)$, then $\det A \neq 0$ since $\det \alpha I \neq 0$, so that $A^{-1} = B \in F_k$ exist. Thus deduce the identity

$$(1) \qquad f(y_1, \cdots, y_r) = \alpha(y_1, \cdots, y_r) \cdot B$$

where $\alpha(y)$ is a scalar function on $(F_k)^r$, not identically zero. Conjugate both sides of (1) by a matrix $D \in F_k$ so that DBD^{-1} is in a Jordan canonical form. Since $f(x)$ is a polynomial,

$$Df(y_1, \cdots, y_r)D^{-1} = f(Dy_1D^{-1}, \cdots, Dy_rD^{-1}) = f(\bar{y}_1, \cdots, \bar{y}_r) .$$

By (1), $Df(y)D^{-1} = \alpha(y)DBD^{-1}$. Since