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TOPOLOGICAL LATTICE ORDERED GROUPS

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Several types of hulls and completions of lattice ordered groups have been obtained by algebraic methods. In this paper is laid some groundwork for the application of topological and uniform-space concepts to the same end by setting forth those links—topological, algebraic and semantic—between a topological lattice ordered group H and a topologically dense ℓ-subgroup G.

In section one a convex ℓ-subgroug of a representable ℓ-group G is proved to be order closed if and only if it is closed with respect to every Hausdorff ℓ-topology. In section three the disjunctive formulas which hold in a topological ℓ-group are proved to be the same as those which hold in a topologically dense ℓ-subgroup. The last section contains the continuous versions of the classical ℓ-group representation theorems.

The list of contributors to the theory of topological lattice ordered groups is long; Redfield gives a historical sketch and a good bibliography in [18]. This investigation makes particularly heavy use of the work of R. H. Madell, that of Redfield, and the ongoing work of B. Smarda. But for G. Otis Kenny, whose work introduced the author to these ideas and who unselfishly participated in many stimulating discussions, and for Stephen H. McCleary, whose penetrating comments improved an earlier version, the author reserves his deepest gratitude.

1. Order aud topological closure.

A topology $\mathscr{T}$ on a lattice ordered group G which makes group and lattice operations continuous will be termed an ℓ-topology. $(G, \mathscr{T})$ is a *topological lattice ordered group* or *$t\ell$-group*. Smarda [20] first characterized an ℓ-topology in terms of the neighborhood filter of the identity.

THEOREM 1.1. *If $\mathscr{B}$ is the neighborhood filter of* 1 *of the $t\ell$-group $(G, \mathscr{T})$, then $\mathscr{B}$ satisfies the following conditions.*

(a) *$\mathscr{B}$ is a normal filter of subsets of G each containing* 1.

(b) *If $B \in \mathscr{B}$ then $B^{-1} \in \mathscr{B}$.*

(c) *If $B \in \mathscr{B}$ then there is some $A \in \mathscr{B}$ such that $A \cdot A \subseteq B$.*

(d) *If $B \in \mathscr{B}$ and if a and b are disjoint members of G then there is some $A \in \mathscr{B}$ with $Aa \wedge Ab \subseteq B$.*

The neighborhood filter of any $g \in G$ is $\mathscr{B}g = g\mathscr{B}$. Conversely, if $\mathscr{B}$ is any filter of subsets of G satisfying (a)~(d) *then by defining*