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ON BANACH SPACES HAVING THE PROPERTY G. L.

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A Banach space E has the property G. L. if every absolutely summing operator defined on E factors through an L_1-space. Some properties of spaces having G. L. property are investigated, using methods of Banach ideals of operators.

1. Introduction and notations. The property G. L. is known to be shared by a number of important classes of Banach spaces: in [6] it is shown that if E'' is isomorphic to a complemented subspace of a Banach lattice (in particular, if E has local unconditional structure in the sense of [4]) then E has the G. L. property. Subspaces of L_1 spaces as well as quotients of $C(K)$ spaces have G. L. property. Moreover, in [17] it is shown that if E is a subspace of a Banach space F *s.t.* $\Pi_2(\mathscr{L}_\infty, F) = \mathscr{L}(\mathscr{L}_\infty, F)$ (in particular if F has cotype 2) and F has the property G. L. then E has the property G. L. In fact, it is easy to see that it is enough for E to be finitely represented in F. In this paper, we try to investigate the property G. L. using methods of Banach ideals of operators. It is shown that this property is characterized by a perfect ideal $[\Gamma, \gamma]$. We obtain a description of the conjugate ideal $[\Gamma^*, \gamma^*]$ and deduce that $[\Gamma, \gamma]$ is a symmetric ideal hence E has G. L. iff E' has it.

It is also shown that a number of properties, known to hold for spaces having *l.u.st.* in the sense of [4] are common to all the spaces having G. L. For example, if E is a space having G. L. which does not contain l_∞^n-*s* uniformly, then either E contains l_1^n-*s* uniformly and uniformly complementably, or E does not contain l_1^n-*s* uniformly at all.

It follows that if E is a space having G. L. and F a Banach space, then there exist compact nonnuclear operators from E to F and from F to E. These are partial generalizations to results of Davis and Johnson (see [2] and [9]). We show also that for spaces having G. L. the property $\Pi_2(\mathscr{L}_\infty, E) = \mathscr{L}(\mathscr{L}_\infty, E)$ implies that E is of cotype 2; we show a dual implication as well.

The paper is divided into two parts. In §2 we describe some tools in Banach ideals of operators; in §3 we use these tools in investigating spaces having G. L. It seems to us that these tools may be useful in other contexts.

The notations are of two kinds:

(1) *General notations.* We use standard notations of Banach