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AN EXPLICIT FORMULA FOR THE FUNDAMENTAL UNITS OF A REAL PURE SEXTIC NUMBER FIELD AND ITS GALOIS CLOSURE

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The object of this paper is to give a set of fundamental units of a real pure sextic number field $K = \boldsymbol{Q}(\sqrt[6]{a^6 - 1})$, where a is a special type of natural number and $a^6 - 1$ is not necessarily 6th power free. It is also shown that a set of fundamental units of the galois closure $L = K(\sqrt{-3})$ of K is formed by a real unit and its conjugates.

Let d be a 6th power free natural number which is not a perfect square or a perfect cube in the rational number field $\boldsymbol{Q}$. Put $\theta = \sqrt[6]{d}$; then $K = \boldsymbol{Q}(\theta)$ is a real pure sextic number field. We investigate the group of units of K for a special type of d as follows. Let d be given by

$$d = c(b^6c \pm 2)(b^{12}c^2 \pm b^6c + 1)(b^{12}c^2 \pm 3b^6c + 3) \tag{1}$$

with natural numbers b and c. Put

$$a = b^6c \pm 1\ . \tag{2}$$

(The $\pm$ signs correspond respectively throughout this paper.) Then

$$b^6d = a^6 - 1 \tag{3}$$

and $K = \boldsymbol{Q}(\sqrt[6]{a^6 - 1})$.

THEOREM 1. *The notation being as above, we assume that $d > 1$ and d is square free. Then*

$$\xi_1 = a - b\theta\ ,\quad \xi_2 = a + b\theta\ ,\quad \xi_3 = a^2 + ab\theta + b^2\theta^2 \tag{4}$$

form a set of fundamental units of K.

As to explicit formulas for the fundamental units of number fields, G. Degert [2] has given one for certain real quadratic fields. As an application of the Jacobi-Perron algorithm (J.P.A.), L. Bernstein, H.-J. Stender and R. J. Rudman has extended Degert's result to certain real pure cubic, quartic and sextic fields (see [9] and [10]). On the other hand, H. Yokoi has given a different formula for the fundamental units of real quadratic and pure cubic number fields in [11], [12] and [13]. Theorem 1 is an extension of Yokoi's result to real pure sextic fields. A similar formula can be