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PEIRCE IDEALS IN JORDAN TRIPLE SYSTEMS

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We show that an ideal in a Peirce space $J_i (i = 1, 1/2, 0)$ of a Jordan triple system J is the Peirce i-component of a global ideal precisely when it is invariant under the multiplications $L(J_{1/2}, J_{1/2})$, $P(J_{1/2})P(J_{1/2})$ (for $i=1$); under $L(J_{1/2}, J_{1/2})$, $P(J_{1/2})P(J_{1/2})$, $P(J_{1/2})P(e)P(J_{1/2})$, $L(J_{1/2}, e)P(J_0, J_{1/2})$ (for $i = 0$); under $L(J_1)$, $L(J_0)$, $L(J_{1/2}, e)L(e, J_{1/2})$, $L(J_{1/2}, e)P(e, J_{1/2})$ (for $i = 1/2$). We use this to show that the sub triple systems J_1 and J_0 are simple when J is. The method of proof closely follows that for Jordan algebras, but requires a detailed development of Peirce relations in Jordan triple systems.

Throughout we consider Jordan triple systems (henceforth abbreviated JTS) with basic product $P(x)y$ linear in y and quadratic in x, with derived trilinear product $\{xyz\} = P(x, z)y = L(x, y)z$, over an arbitrary ring Φ of scalars. Because we are already overburdened with subscripts and indices, we prefer not to treat the general case of Jordan pairs directly, but rather derive it via hermitian JTS. For basic facts about JTS and Jordan pairs we refer to [1], [3], [6]. Our analysis of Peirce ideals will closely follow that for Jordan algebras; although the basic lines of our treatment are the same as in [4], the triple system case requires such horrible computations that we do not carry out so fine an analysis, but concentrate just on the main simplicity theorem.

1. Peirce relations in Jordan triple systems. Any Jordan triple system satisfies the general identities

$$\text{(JT1)} \quad L(x, y)P(x) = P(x)L(y, x)$$

$$\text{(JT2)} \quad L(x, P(y)x) = L(P(x)y, y)$$

$$\text{(JT3)} \quad P(P(x)y) = P(x)P(y)P(x)$$

and the linearization

$$\text{(JT3')} \quad P(\{xyz\}) + P(P(x)y, P(z)y) = P(x)P(y)P(z) + P(z)P(y)P(x) + P(x, z)P(y)P(x, z) .$$

A more useful version of this is the identity

$$\text{(JT4)} \quad P(\{xyz\}) = P(x)P(y)P(z) + P(z)P(y)P(x) + L(x, y)P(z)L(y, x) - P(P(x)P(y)z, z) .$$

Other basic identities we require are