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ANNIHILATION OF IDEALS IN COMMUTATIVE RINGS

JAMES A. HUCKABA AND JAMES M. KELLER

Four theorem are proved concerning the annihilation of finitely generated ideals contained in the set of zero divisors of a commutative ring.

1. Introduction. An important theorem in commutative ring theory is that if I is an ideal in a Noetherian ring and if I consists entirely of zero divisors, then the annihilator of I is nonzero. This result fails for some non-Noetherian rings, even if the ideal I is finitely generated. We say that a commutative ring R has *Property* (A) if every finitely generated ideal of R consisting entirely of zero divisors has nonzero annihilator. Property (A) was originally studied by Y. Quentel in [7]. (Our Property (A) is Quentel's Condition (C).) Theorem 1 shows that all nontrivial graded rings have Property (A). (For our purposes a *nontrivial graded ring* is a ring R graded over the integers such that R contains an element x, not a zero divisor, of positive homogenous degree.) Theorem 2 completely characterizes those reduced rings with Property (A).

Property (A) is closely connected with two other conditions on a reduced ring. One is the *annihilator condition* (a.c.): If (a, b) is an ideal of R, then there exists $c \in R$ such that $\mathrm{Ann}(a, b) = \mathrm{Ann}(c)$. The other condition is that $\mathrm{MIN}(R)$, the space of minimal prime ideals of R, is compact. Our Theorem 3 shows that for a reduced coherent ring R Property (A), (a.c.), and the total quotient ring of R being a von Neumann regular ring are equivalent conditions; and that each (and hence all) of these conditions imply that $\mathrm{MIN}(R)$ is compact. Finally, in Theorem 4, we prove that every reduced nontrivial graded ring satisfies (a.c.).

We assume that all rings are commutative with identity. If R is such a ring, let $T(R)$ be the total quotient ring of R, let $Z(R)$ be the set of zero divisors of R, and let $Q(R)$ denote the complete ring of quotients of R as defined in [5]. Elements of R that are not zero divisors are called *regular elements*.

2. Graded rings.. Y. Quentel, [7, p. 269], proved that if R is a reduced ring, then the polynomial ring $R[X]$ satisfies Property (A). We generalize this to arbitrary nontrivial graded rings, and hence to polynomial rings that are not necessarily reduced.

THEOREM 1. *If R is nontrivial graded ring, then R satisfies Property* (A).