PACIFIC JOURNAL OF MATHEMATICS
Vol. 83, No. 2, 1979

ON THE NONOSCILLATION OF PERTURBED FUNCTIONAL DIFFERENTIAL EQUATIONS

JOHN R. GRAEF, YUICHI KITAMURA, TAKAŜI KUSANO, HIROSHI ONOSE, AND PAUL W. SPIKES

We study the behavior of the solutions of the second order nonlinear functional differential equation

$$(1)\qquad (a(t)x')' = f(t, x(t), x(g(t)))$$

where a, g: $[t_0, \infty) \to R$ **and** f: $[t_0, \infty) \times R^2 \to R$ **are continuous,** $a(t) > 0$, **and** $g(t) \to \infty$ **as** $t \to \infty$. **We are primarily interested in obtaining conditions which ensure that certain types of solutions of (1) are nonoscillatory. Conditions which guarantee that oscillatory solutions of (1) converge to zero as** $t \to \infty$ **are also given. We apply these results to the equation**

$$(2)\qquad (a(t)x')' + q(t)r(x(g(t))) = e(t, x)$$

where q: $[t_0, \infty) \to R$, r: $R \to R$, e: $[t_0, \infty) \times R \to R$ **are continuous and** a **and** g **are as above. We compare our results to those obtained by others. Specific examples are included.**

In the case of nonlinear ordinary equations, the search for sufficient conditions for all solutions to be nonoscillatory has been successful; see, for example, the papers of Graef and Spikes [4-7], Singh [11], Staikos and Philos [14], and the references contained therein. The only such results known for functional equations to date are due to Graef [3], Kusano and Onose [9], and Singh [13]. Moreover, none of the results in [3], [9], or [13] apply to equation (2) if $e(t, x) \equiv 0$ or if r is superlinear, e.g., $r(x) = x^\gamma$, $\gamma > 1$. We refer the reader to the recent paper of Kartsatos [8] for a survey of known results on the oscillatory and asymptotic behavior of solutions of (1) and (2).

In view of a recent paper by Brands [1], it does not appear to be possible to obtain integral conditions on $q(t)$ which will guarantee that all solutions of (2) with $e(t, x) \equiv 0$ are nonoscillatory and which are similar to those usually encountered in the study of ordinary equations. (We will return to this point again in §2.) So too our main results in this direction when applied to equation (2) require that $e(t, x) \not\equiv 0$ (cf. conditions (27) and (28)). Although all the results presented here hold if $r(x)$ is sublinear, we are especially interested in the superlinear case.

2. Main results. The results in this paper pertain only to the continuable solutions of (1). A solution $x(t)$ of (1) will be called