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# THERE ARE $2^c$ NONHOMEOMORPHIC CONTINUA IN $\beta R^n - R^n$

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**In this paper it is shown that for $n \geqq 3, \beta R^n - R^n$ contains $2^c$ nonhomeomorphic continua. In the proof we will also construct $c$ continua in $\beta R^3 - R^3$ with nonisomorphic first Cech cohomology groups and $2^c$ compacta in $\beta R^3 - R^3$ no two of which have the same shape.**

Introduction. Much work has been done in the study of the Stone-Čech compactification of the natural numbers. Some of these results have been applied to the study of $\beta X - X$ for other topological spaces $X$, as in the proof of Frolik's result that $\beta X - X$ is not homogeneous for a nonpseudocompact space $X$ (see [9]). Shape theory has offered new methods for examining $\beta X$ and $\beta X - X$ that utilize the intrinsic topological properties of $\beta X$, as is illustrated in this paper in the case of $\beta R^n$. Using the fact that shape factors through Čech cohomology, we will construct $c$ continua in $\beta R^3 - R^3$, no two of which have the same shape. Then, a particular embedding of subsets of the continua into $\beta R^3$ will exhibit $2^c$ compacta in $\beta R^3 - R^3$ with different shapes. An easy modification of the compacta will yield $2^c$ nonhomeomorphic continua in $\beta R^3 - R^3$, the proof of which utilizes the properties of shape dimension as developed by J. Keesling [5]. From this it follows that for $n \geqq 3$ there are $2^c$ nonhomeomorphic continua in $\beta R^n - R^n$.

Preliminaries. Let $\beta X$ denote the Stone-Čech compactification of a space $X$. For references, see Gillman and Jerison [2], or Walker [9]. $H^n(X)$ will denote the $n$-dimensional Čech cohomology of $X$ with coefficients in $Z$ based on the numerable covers of $X$. Also, $[X, S^1]$ will denote all homotopy classes of maps from $X$ into $S^1$, with the group structure induced by the group structure on $S^1$. Since $S^1$ is a $K(Z, 1)$, $H^1(X)$ is isomorphic to $[X, S^1]$. Finally, let $\widetilde{\prod} A_i$ be the group $\prod_{i \in Z} A_i / \sum_{i \in Z} A_i$.

The following theorems will be used in this paper:

THEOREM 1 (*Lemma* 1.7 *of* [1]). *For $X$ normal and connected, there is an exact sequence $0 \to C(X)/C^*(X) \to [\beta X, S^1] \to [X, S^1] \to 0$ where $C(X)$ is the additive group of real valued continuous functions on $X$, and $C^*(X)$ is the subgroup of bounded real continuous functions.*

THEOREM 2 (*Theorem* 1.6 *of* [5]). *Let $n \geqq 1$ be an integer. Let*