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NONOPENNESS OF THE SET OF THOM-BOARDMAN MAPS

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1. Introduction. In this paper, we show that the set of all C^∞ Thom-Boardman maps from an n-dimensional manifold is not open iff corank two singularities occur generically. The latter is known to occur iff either $n \leqq p$ and $2p \leqq 3n-4$ or $n > p$ and $2p \geqq n+4$. In the course of the proof, we establish a variation of Mather's Multitransversality Theorem: we show that jets have extensions which are multitransverse to given submanifolds of the jet bundle except possibly at the original jet. As an application of this extension theorem, we show that, in Mather's "nice range of dimensions," each jet z has a representative f ($z = j^k f(x)$) such that f is infinitesimally stable on a deleted neighborhood of x.

First we recall some properties of Thom-Boardman singularities; for more details, the reader is referred to [1], [11] and [3]. In this paper N (respectively P) will always be an n (respectively p) dimensional manifold without boundary. There is a finite partition of the jet bundle $J^k(N, P)$ into embedded submanifolds S^I, called *Thom-Boardman Singularities*, each I a nonincreasing sequence of nonnegative integers. Consider f in $C(N, P)$, the set of smooth maps from N to P. Let $S^I(f)$ denote $(j^k f)^{-1}S^I$, $j^k f$ the jet extension of f. Then $j^k f \pitchfork S^i$ implies that $S^i(f)$ is the set of points at which $\dim \ker Tf = i$; $j^k f \pitchfork S^{i,j}$ implies $j^k f \pitchfork S^i$ and $S^{i,j}(f) = S^j(f \mid S^i(f))$, etc. We call f a *Thom-Boardman map* if $j^k f \pitchfork S^I$ for all I, for all k. By Thom's Transversality Theorem, the set of Thom-Boardman maps is residual (i.e., is a countable intersection of open, dense sets), hence is dense, in the Whitney C^∞ topology.

THEOREM 1.1. *The Thom-Boardman maps form an open subset of $C(N, P)$ iff either $n \leqq p$ and $2p > 3n-4$ or $n > p$ and $2p < n+4$.*

Let $r = \max(n-p, 0)$, let S_i denote S^{i+r} (jets of corank i), let $S_{i,j}$ denote $S^{i+r,j}$, etc. The condition on n and p in the theorem is precisely the condition that the codimension (abbreviated cod) of S_2 be greater than n, hence that maps cannot take on S_2 singularities transversally.

If $\operatorname{cod} S_2$ is greater than n, then a map is Thom-Boardman iff it is transverse to all Morin singularities (which are the $S_{1;k}$, where $1;k$ means $1, \cdots, 1, 0$, 1 occurring k times). In this case, a map is