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EXTENSION OF ACTIONS ON STIEFEL MANIFOLDS

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It is natural to ask for examples of π-biaxial actions in the unitary and simplectic case which do not come from the orthogonal case. Here we provide examples of such actions.

Introduction. Let us consider the left translation action of $U(n)(Sp(n))$ on the Stiefel manifold $U(n+k+2)/U(n+k)$ $(Sp(n+k+2)/Sp(n+k))$, k nonnegative integer, $n \geqq 2$.

The main result to be proved here is that the above action can not be extended to a biaxial $O(2n)(U(2n))$ action. The proof uses strongly the correspondence between $U(n)(Sp(n))$ π-biaxial manifolds with orbit space diffeomorphic to a disk and framed submanifolds of the sphere.

The main references for this article are the book Introduction to Compact Transformation Groups (Bredon [3]) for the general theory of groups actions and the mimeographed notes (Bredon [4]) Biaxial Actions of the Classical Groups for the classification of such actions and characterization of restrictions of π-biaxial manifolds.

1. Preparatory material. Let G be a compact Lie group and $\sigma: G \to Gl(V)$ be a representation of G on the real vector space V. By a G-manifold "modeled on σ" we mean a smooth G manifold such that each orbit in M has an open invariant neighborhood which is equivariantly diffeomorphic to an open invariant set in the representation space V of σ.

Let $d = 1, 2, 4$. In these three cases we let G_n^d stand for $O(n)$, $U(n)$ or $Sp(n)$. The standard representation of G_n^d on R^{nd} will be denoted by σ_n and the trivial real k-dimensional representation by θ_k. A G-manifold M is modeled on $2\sigma_n + \theta_k$ $k < 0$ if $M \times R^{-k}$ is modeled on $2\sigma_n(G_n^d$ acts trivially on $R^{-k})$.

DEFINITION 1.1. A G_n^d manifold M, $n \geqq 2$, will be called biaxial if it is modeled on the representation $2\sigma_n + \theta_k$. It is not hard to see that a G_n^d manifold is biaxial iff the following four conditions hold:

1. The principal orbit type is G_n^d/G_{n-2}^d. The other orbit types (if any) are G_n^d/G_{n-1}^d and fixed points (if any).
2. The representation of G_n^d about a fixed point is $2\sigma_n + \theta_k$.
3. The slice representation of G_{n-1}^d on the normal space to an orbit at a point with isotropy group G_{n-1}^d is $\sigma_{n-1} + \theta_{k+d+1}$.