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STOCHASTIC DIFFUSION ON AN UNBOUNDED DOMAIN

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In this paper we study a stochastic partial equation of the following form.

$$\frac{\partial u}{\partial t} = 1/2\frac{\partial^2 u}{\partial x^2} - f(u) + \alpha(x, t)$$

where f is a monotone nonlinear operator and α is a "white noise" process in x and t. In a previous paper we demonstrated the existence of a unique solution in a generalized sense for x in a *bounded* domain. This solution was decomposed into the sum of a stationary process and a transient process. An explicit representation was found for the stationary distribution of the stationary process. If f is an ordinary function of $u(x)$ then the stationary distribution is associated with a Markov process in x. The purpose of this paper is to remove the restriction of boundedness for the bounded domain.

The motivation for this study was to establish a link between stochastic partial differential equations and constructive quantum field theory. The basic idea is that the stationary distributions of certain stochastic partial differential equations will be Euclidean Markov fields. See Nelson [3]. For an example see Appendix.

1. Definitions. The equation studied is formally

$$(1) \qquad u_t(x, t) = \frac{1}{2}u_{xx}(x, t) - \lambda^2 u(x, t) - f(u(x, t)) + \alpha(x, t)$$

$$(x \varepsilon (-\infty, +\infty), \lambda > 0)$$

and for convenience $u(x, 0) = 0$, $\alpha(x, t)$ is a "white noise" process i.e., $E(\alpha(x, t)\alpha(y, s)) = \delta(x - y)\delta(t - s)$.

Converting (1) to an integral equation

$$(2) \qquad u(x, t) = -\int_0^t\int_{-\infty}^{+\infty} G_\lambda(t - s, x, y)f(u(y, s))dyds + W(x, t)$$

with

$$G_\lambda(t - s, x, y) = \exp(-\lambda^2 t - (x - y)^2/2(t - s))/\sqrt{2\pi(t - s)}\ .$$

$W(x, t)$ is a Gaussian process with mean 0 and covariance equal to

$$E(W(x, t)W(y, s)) = \int_0^{\min(t,s)} G_\lambda(t + s - 2r, x, y)dr$$

formally