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2-FACTORIZATION IN FINITE GROUPS

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Let G be a finite group, and S be a nonidentity 2-subgroup of G. Then, it is naturally conjectured that there exists a nonidentity $N_G(S)$-invariant subgroup of S, whose normalizer contains all the subgroups H of G with the following properties: $(\alpha)S$ is a Sylow 2-subgroup of H; $(\beta)H$ does not involve the symmetric group of degree four; and $(\gamma)C_H(O_2(H))\subseteq O_2(H)$. The purpose of this paper is to give a partial answer to this problem.

1. Introduction. Suppose π is a set of primes, and X is a finite group. Let $\mathscr{D}(X:\pi)$ be the family of all groups D that are involved in X with the following properties; $(\alpha)D$ possesses a normal simple subgroup E, $(\beta)C_D(E) \subseteqq E$ (that is, D/C induces outer automorphisms of E), and $(\gamma)D/E$ involves a dihedral group of order $2p$ for some prime $p(\geqq 5)$ in π.

THEOREM. *Let π be a set of primes. Suppose G is a finite group, and S is a nonidentity 2-subgroup of G. Assume that for any nonidentity subgroup T of S which is normal in $N_G(S)$,*

(1) *S is normal in some Sylow 2-subgroup of $N_G(T)$; and*

(2) *$\mathscr{D}(N_G(T)/G_G(T):\pi) = \phi$.*

Then there exists a nonidentity subgroup $W(S)$ of S which satisfies the following conditions (a) *and* (b):

(a) *$W(S)$ is normal in $N_G(S)$; and*

(b) *$W(S)O(H)$ is normal in H for any solvable π-subgroup H of G which satisfies the following conditions (α) and (β):*

(α) *S is a Sylow 2-subgroup of H: and*

(β) *H is S^4-free, where S^4 denotes the symmetric group of degree four.*

REMARK 1.1. The condition (1) of the theorem is satisfied, whenever S is normal in some Sylow 2-subgroup of G.

In general, suppose p is a prime, G is a finite group, and S is a nonidentity p-subgroup of G. Let $Qd(G, S)$ be the family of all subgroups H of G that satisfy the following conditions: (α) S is a Sylow p-subgroup of H; and $(\beta)H$ is p-constrained, and p-stable (if $p = 2$, S^4-free). Then, what are the relations among the elements of $Qd(G, S)$? Furthermore, what are the relations between G and the elements of $Qd(G, S)$? These problems were proposed by G. Glauberman and J. G. Thompson, and for which amazing progresses