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COHOMOLOGY OVER BANACH CROSSED PRODUCTS. APPLICATION TO BOUNDED DERIVATIONS AND CROSSED HOMOMORPHISMS

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The purpose of this work is to study the structure of bounded derivations and crossed homomorphisms of the Banach crossed product $\mathfrak{A}=L^1(G, A)$ of a Banach-*-algebra A acted upon by a locally compact group G. As bounded derivations and crossed homomorphisms are related to 1-cocycles, we first define and study cohomology over $\mathfrak{A}$, generalizing cohomology over group algebras. Then, if G is amenable and A is a C^*-algebra, or the dual of a Banach space, we show that a bounded derivation (resp. a crossed homomorphism) on $\mathfrak{A}$ is equivalent to some couple of a bounded derivation (resp. a crossed homomorphism) from A to $M_1(G, A)$ and a bounded measure on A with value in the centralizers of A (resp. an element of $\mathfrak{A}$).

1. Introduction. Crossed products of Banach algebras and locally compact groups are interesting objects from a mathematical point of view because they are generalizations of group algebras, from a physical point of view because they are useful tools in describing quantum dynamical systems. Hence it would be interesting to know the structure of their automorphisms and derivations. For a large class of automorphisms, the answer is given in [2]. In this paper, our aim is to begin the study of bounded derivations and crossed homomorphisms of Banach crossed products. For that purpose, cohomology techniques seem to be useful and this is the reason why we will begin with cohomology over Banach crossed products, a generalization of cohomology over group algebras worked out in [15].

Given a locally compact group G acting on a Banach $*$-algebra A, $\mathfrak{A} = L^1(G, A)$ will be the Banach crossed product of these two objects. In paragraph 2, we collect known results about centralizers on A and vector measures, and define several module structures on them in paragraph 3. Paragraph 4 is devoted to the definition of cohomology over $\mathfrak{A}$, while paragraph 5 contains a Riesz representation theorem for the elements of the spaces introduced in the preceding paragraph. In paragraph 6 we extend the cohomology over $\mathfrak{A}$ to its centralizers. Finally paragraph 8 characterizes the structure of derivations and crossed homomorphisms, using the notion of vector means developed in paragraph 7.