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SPHERICAL MEAN PERIODIC FUNCTIONS ON SEMI SIMPLE LIE GROUPS

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Let G be a connected semisimple noncompact Lie group with finite center. We define the notion of a smooth spherical mean periodic function (with respect to a fixed maximal compact subgroup K of G) and show that the classical results of L. Schwartz for mean periodic functions on the real line hold in this context.

1. Introduction. The study of mean periodic functions started with Delsarte ([1]) who was interested in solving the convolution equation

$$\mu * f = 0$$

where μ is a measure of compact support on $\boldsymbol{R}$ and f a continuous function on $\boldsymbol{R}$. He was able to show that under certain conditions a general solution f can be written as a linear combination of "exponential monomial" solutions of the above equation. A mean periodic function on $\boldsymbol{R}$ is a continuous function f satisfying the above convolution equation for a nontrivial μ. In his famous paper ([11]) L. Schwartz studied mean periodic functions in detail, introduced the notion of the spectrum of a mean periodic function and showed that a mean periodic function f can be approximated by finite linear combinations of the functions in the spectrum of f. Malgrange in [10] studied the case of mean periodic functions on $\boldsymbol{R}^n$ for $n > 1$ and showed that a weaker version of Schwartz's result holds in this case.

The study of smooth mean periodic functions for the group SL $(2, \boldsymbol{R})$ was taken up by Ehrenpreis and Mautner in [4], [5] and results analogous to those of Schwartz were obtained by them. Since then harmonic analysis of spherical functions on semisimple Lie groups has been studied extensively ([2], [6], [7], [9], [12]).

The purpose of this paper is to use these powerful results along with the original results of Schwartz and Malgrange to study the case of spherical mean periodic functions on a noncompact semisimple Lie group G with finite center.

2. Preliminaries. Throughout § 2 and § 3 G will denote a noncompact semisimple Lie group with finite center and of real rank 1, K a fixed maximal compact subgroup of G and $\boldsymbol{R}$ the real line. Any unexplained terminology in this section can be found in [8].