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# STABILITY CONDITIONS FOR NONLINEAR PRODUCTS AND SEMIGROUPS

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**Let $D$ be a subset of a Banach space. Suppose $R(t)$: $D \to D (t \geqq 0)$ is an "almost-semigroup," in the sense that $R(t)R(s)$ is close to $R(t+s)$. If $R$ also satisfies certain stability conditions, then $R(t/n)^n$ converges to some semigroup $S(t)$ as $n \to \infty$. The stability conditions are motivated by several examples involving nonlinear partial differential equations.**

**1. Introduction.** Let $D$ be a subset of a Banach space. By a *transformation* on $D$ we shall mean a one-parameter family of mappings

$$S(t): D \longrightarrow D \quad (t \geqq 0)$$

(not necessarily linear or continuous), such that

$$S(0)f = f \quad \text{for all } f \in D .$$

$S$ is a *semigroup* if in addition

$$S(t + s)f = S(t)S(s)f \quad (t, s \geqq 0;\ f \in D) .$$

(This notation is essentially that of [4]. Numerous variants exist. For instance, in [3], such an $S$ is referred to as a "semiflow"; the term "semigroup" is reserved for the linear case in that paper.)

In this paper and in [9], we shall consider the following problem: Let $S_1$ and $S_2$ be semigroups on $D$. Give sufficient conditions for the existence of a semigroup $S$ on $D$ such that

$$S(t)f = \lim_{n \to \infty} [S_1(t/n)S_2(t/n)]^n f \quad (t \geqq 0, f \in D) . \tag{1.1}$$

Equation (1.1) is commonly known as the *Trotter product formula*, because of Trotter's results for linear semigroups in [12].

The product problem generalizes as follows: Investigate the conditions under which a transformation $R$ and a semigroup $S$ satisfy

$$S(t)f = \lim_{n \to \infty} R(t/n)^n f \quad (t \geqq 0, f \in D) . \tag{1.2}$$

The Lax Equivalence Theorem of numerical analysis takes the form (1.2) with $R$ and $S$ linear; see [5]. The Crandall-Liggett exponential formula for nonlinear semigroups is of the form (1.2) with $R(t)$ equal to $(I + tA)^{-1}$, the resolvent of an operator; see [4]. Equation