PACIFIC JOURNAL OF MATHEMATICS
Vol. 85, No. 1, 1979

SEMILATTICES HAVING BIALGEBRAIC CONGRUENCE LATTICES

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The congruence lattice of a semilattice is algebraic (=compactly generated). In this paper those semilattices having bialgebraic congruence lattices are characterized. We are also able to characterize those semilattices having the stronger property that their congruence lattices support a compact, Hausdorff topology which makes them into topological lattices.

Let $\mathscr{S}$ be the category of (meet) semilattices and meet-preserv ing maps. A congruence on a semilattice S is a subset of $S \times S$ which is both a subsemilattice and an equivalence relation. When ordered by inclusion $\Theta(S)$, the set of all congruences on S, becomes a coatomically generated (i.e., every element is an infimum of co-atoms), algebraic (=compactly generated) lattice (cf. [3], [8]). As such it will have very strong completeness or topological properties examplified by the fact that it supports a naturally defined topology relative to which it becomes a compact topological semilattice. In this paper we are concerned with those semilattices whose congruence lattices have even stronger topological properties. First, we are able to characterize, both externally and internally, those semilattices S for which $\Theta(S)$ is bialgebraic (i.e., both $\Theta(S)$ and its dual are algebraic). The external characterization is based upon whether S has either of two very elementary semilattices as a quotient.

Bialgebraic lattices will then support two naturally defined topologies. Meet is continuous relative to one and join is continuous relative to the other. In general, these two topologies do not agree. However, in many interesting cases, such as the lattice of all subsets of a set, they do. Bialgebraic lattices in which these topologies coincide will be called coordinated. In § 3 we characterize those semilattices having coordinated bialgebraic congruence lattices. Again, both internal and external characterizations are given. For the external characterization we need only add a third familiar semilattice to the two needed for the bialgebraic situation.

The theory of algebraic lattices is fully developed in the book by Crawley and Dilworth [2] where they are called compactly-generated lattices. Basic information about congruence lattices on semilattices is to be found in the papers of Dean and Oehmke [3] and Papert [8]. Kent and Atherton have previously discussed