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SOME CLASSES OF RINGS WITH INVOLUTION SATISFYING THE STANDARD POLYNOMIAL OF DEGREE 4

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The identities valid in the ring of real quaternions are defined in more general classes of rings with involution. With one exception, these classes of rings satisfy the standard polynomial of degree 4 and form a chain under inclusion. There are examples which show that these inclusions are proper. An example of an exterior algebra shows that a ring with involution whose symmetric elements commute does not necessarily satisfy the standard polynomial of degree 4.

We assume that a ring has a unit although some of the proofs which follow do not require the existence of a unit in general.

Throughout the paper, R will be a ring equipped with an involution $*$, i.e., a map $R \to R$ such that for all $x, y \in R$, $(x + y)^* = x^* + y^*$, $(xy)^* = y^*x^*$ and $x^{**} = x$.

The sets S and K of *symmetric* and *skew-symmetric* elements of R consist respectively of elements x of R such that $x^* = x$ and $x^* = -x$.

The *trace* and *norm* of an element x in R are respectively $T(x) = x + x^*$ and $N(x) = xx^*$.

As usual, $[x, y] = xy - yx$ denotes the commutator, i.e., the standard polynomial of degree 2 of $x, y \in R$ and the symbol Z denotes the center of R.

We shall require that a ring R be subject to certain identities, all of which are valid in the case of real quaternions. In other words, we are extending the properties of real quaternions to more general classes of rings.

R will be called a *scalar product ring* if for all $x, y \in R$,

$$T(xy) = T(yx) . \tag{1}$$

This definition follows from Dyson [1].

R is called a *normal ring* if for all $x \in R$, $xx^* = x^*x$.

R is called a *central trace ring* if for all $x \in R$, $T(x) \in Z$.

R is called a *central norm ring* if for all $x \in R$, $N(x) \in Z$.

R is called a *central symmetric ring* if the symmetric elements of R are central i.e., $S \subset Z$.

R is called a *commuting symmetric ring* if the symmetric elements of R commute.