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MODELLING EXPANSION IN REAL FLOWS

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Dedicated to the memory of Rufus Bowen

We show that any real flow without fixed points is the homomorphic image of a suspension of the shift on a bisequence space and the homomorphism is one-to-one between invariant residual sets. If the original flow is one-dimensional this homomorphism is an isomorphism. We then use this model of a real flow to lift $\mathscr{F}$-expansiveness for any class $\mathscr{F}$ of continuous functions from the reals into the reals fixing zero, and thus generalize the results of Bowen and Walters [2]. Various other properties of the suspension model are discussed.

0. Introduction. In [2] Bowen and Walters introduced the concept of expansiveness for real flows relative to the class $\mathscr{C}$ of all continuous functions from the reals into the reals which fix zero. In [5] this concept was extended to arbitrary transformation groups and to arbitrary classes $\mathscr{F}$ of continuous functions from the acting group into itself which fixes the group identity. It is well-known (see [4]) that any expansive discrete flow can be lifted to a subshift and the authors of [2] managed to obtain an analogous result for real flows: every $\mathscr{C}$-expansive real flow can be lifted to the suspension of a shift on a symbol space. Results of this type are also obtained for $\mathscr{F}$-expansive discrete flows in [5] but no results were given for other transformation groups. The main direction of this paper is to extend the methods and results of [2] to show that every fixed point free $\mathscr{F}$-expansive real flow can be lifted to a suspension of a shift on a bisequence space (of course, not a finite symbol space in general) which is also $\mathscr{F}$-expansive. This generalizes the Bowen and Walters result and also covers situations where real flows can not be $\mathscr{C}$-expansive but are expansive for subclasses $\mathscr{F}$, such as certain real flows on a 2-torus (see [5] and [3] for details).

Since the model that we obtain is independent of any expansive properties of the original flow, we also obtain a result of interest in its own right concerning real flows and suspensions. Birkhoff [1] pointed out that certain dynamical systems have a global cross section and can thus be regarded as real suspensions of discrete flows. Schwartzman [6] showed that for real flows on compact metric spaces, the property of possessing a global section was equivalent to the flow being a suspension over a discrete flow on that