PACIFIC JOURNAL OF MATHEMATICS
Vol. 85, No. 1, 1979

THE GROUP OF UNITS OF A COMMUTATIVE SEMIGROUP RING

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We seek a characterization, in terms of the coefficients and support of an element, of the units of the semigroup ring $R[X; S]$, where R is a commutative ring with identity and S is an additive abelian semigroup with identity. Such a characterization requires some restrictions on the semigroup S.

We obtain results of the desired form in §2 for the case where S is torsion-free and cancellative, and in §3 under the weaker hypothesis that S is torsion-free and has no nonzero idempotents. Under this weaker hypothesis on S, the torsion subgroup of the group of units of $R[X; S]$ is determined in §4 of this paper.

1. Introduction. If R is a commutative ring with identity, then necessary and sufficient conditions are known in order that a polynomial $f \in R[\{X_\lambda\}]$ should be a unit. Namely, f is a unit of $R[\{X_\lambda\}]$ if and only if the constant term of f is a unit of R and each other coefficient of f is nilpotent [10, p. 683]. In this paper we extend the preceding results by considering the group of units of the semigroup ring $R[X; S]$, where S is a torsion-free additive abelian semigroup with zero (the polynomial ring $R[\{X_\lambda\}_{\lambda \in \Lambda}]$ is isomorphic to the semigroup ring $R[X; \sum_{\lambda \in \Lambda} \oplus Z_\lambda]$, where each Z_λ is the additive semigroup of nonnegative integers). Our main results concerning units of $R[X: S]$ are Theorems 2.4 and 3.2. Before stating these results, we indicate some conventions, terminology, and notation.

All rings considered are assumed to be commutative and to contain an identity element. Semigroups are assumed to be commutative, and we write the semigroup operation as addition; to indicate this, we frequently write "let $(S, +)$ be a semigroup". A semigroup with identity is called a *monoid*. Most of the semigroups we deal with are assumed to be monoids. If R is a ring and S is a semigroup, then we follow the notation of Northcott in [6, p. 128] in writing $R[X; S]$ for the semigroup ring of S over R and in considering the elements of R as 'polynomials' $r_1X^{s_1} + r_2X^{s_2} + \cdots + r_nX^{s_n}$ in X with coefficients in R and exponents in S. If f is a nonzero element of $R[X; S]$, then a representation of the preceding form, where $s_1, \cdots, s_n$ are distinct and each r_i is nonzero, is called the *canonical form* of f, and $\{s_i\}_{i=1}^n$ is called the *support* of f. A unit of $R[X; S]$ with only one element in its support is called a *trivial unit*; such a