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INTEGRAL COMPARISON THEOREMS FOR THIRD ORDER LINEAR DIFFERENTIAL EQUATIONS

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By means of a change of variable in the Riccati equation corresponding to the third order linear equation $Ly \equiv y''' + p(t)y' + q(t)y = 0$ a nonlinear integral equation is obtained which has a solution obtainable by successive approximations under certain conditions on p and q. This technique allows one to obtain new sharp comparison theorems for $Ly = 0$. Several examples are given to illustrate the results.

Introduction. The third linear differential equation

$$Ly \equiv y''' + p(t)y' + q(t)y = 0 \tag{1.1}$$

where $p, q \in C[a, b)$, $0 < a < b \leqq +\infty$, has a very extensive literature relating to the oscillatory and asymptotic behavior of its solutions, with much of the recent impetus coming from the work of Hanan [7], Lazer [10], Azbelev and Caljuk [1], and Barrett [2]. (See also Swanson [12].) In this paper a technique for obtaining some new sharp comparison theorems for (1.1) and a related equation

$$L_1 y \equiv y''' + p_1(t)y' + q_1(t)y = 0 \,, \quad p_1, q_1 \in C[a, b) \tag{1.2}$$

will be introduced. Section 2 below will be devoted to some theorems which are consequences of more general results to be proved in §3. A comparison of the results obtained and their sharpness will also be discussed and illustrated by several examples.

2. Recall that equation (1.1) is said to be disconjugate on an interval I in case no nontrivial solution has more than two zeros on I, counting multiplicity. If $I = [a, +\infty)$(or $(a, +\infty)$), then (1.1) is said to be oscillatory if it has at least one nontrivial oscillatory solution (i.e., a solution with an infinite number of zeros) and nonoscillatory iff all of its solutions are nonoscillatory (i.e., have finitely many zeros).

A useful comparison equation for third order linear equations is the Euler equation

$$y''' + \alpha t^{-2} y' + \beta t^{-3} y = 0 \,, \quad \alpha, \beta \text{ real constants} . \tag{2.1}$$

It is known (cf. [12]) that (2.1) is disconjugate on $(0, +\infty)$ iff $\alpha \leqq 1$ and $|\alpha + \beta| \leqq 2((1 - \alpha)/3)^{3/2}$. There are various tests for oscillation and disconjugacy using the Euler equation in conjunction with known