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DERIVATIONS AND COMMUTATIVITY OF RINGS II

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Let R be a ring with center C, and $\mathscr{I}$ be the additive group of all inner derivations of R. An additive group $\mathscr{D}$ of derivations of R is said to be a primary class of derivations of R if (i) for any $\partial \in \mathscr{D}$ and $\delta \in \mathscr{I}$, $[\partial, \delta] \in \mathscr{D}$, (ii) for any $x \in R$, $\partial x = 0$ for all $\partial \in \mathscr{D}$ if and only if $x \in C$, and (iii) for any prime ideal P in R and any $x \in R$, $\partial x \in P$ for all $\partial \in \mathscr{D}$ if and only if $\delta x \in P$ for all $\delta \in \mathscr{I}$.

Suppose R has a primary class $\mathscr{D}$ of derivations. First we assume, for each $x \in R$ and $\partial \in \mathscr{D}$, there is a $p \in R$ such that $\partial x - (\partial x)^2 p \in C$. Then all nilpotent elements in R form an ideal N of R and R/N is a subdirect sum of division rings and commutative rings. If R is prime, then R has no nonzero divisors of zero. Next, we assume that, for each $x \in R$ and $\partial \in \mathscr{D}$, there is a polynomial $p(t)$ of t with integral coefficients such that $\partial x - (\partial x)^2 p(\partial x) \in C$ or, for each $x \in R$ and $\partial \in \mathscr{D}$, there is a $p \in C$ such that $\partial x - (\partial x)^2 p \in C$. Then $\partial x \in C$ for all $x \in R$ and $\partial \in \mathscr{D}$. If R is prime, then R is necessarily commutative.

1. Introduction. In a previous paper [1], the authors extended the concept of inner derivation to the concept of primary class of derivations for rings and generalized several commutativity theorems given by Wedderburn, Jacobson, Kaplansky, Herstein, Ligh, Putcha, Wilson and Yaqub. Let R be a ring having a primary class $\mathscr{D}$ of derivations whose definitions and basic properties will be recalled later. Among others, the following results were proved:

(1) Suppose, for each $x \in R$ and $\partial \in \mathscr{D}$, there is a $p \in R$ which depends upon x and ∂, such that $\partial x = (\partial x)^2 p$. Then the nilpotent elements in R form an ideal N in R, and R/N is a subdirect sum of division rings and commutative rings.

(2) Suppose, for each $x \in R$, $\partial \in \mathscr{D}$ such a p is a polynomial of ∂x with integral coefficients. Then R is commutative.

(3) Suppose, for each $x \in R$, $\partial \in \mathscr{D}$ such a p described in (1) is central. Then R is a commutative.

The purpose of this paper is to generalize these results further by relaxing the condition "$\partial x = (\partial x)^2 p$." We will consider the condition "$\partial x - (\partial x)^2 p \in C$, the center of R" instead. More precisely we will consider rings R having a primary class of derivations which satisfies one of the following conditions: