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UNBOUNDED MULTIPLIERS ON COMMUTATIVE BANACH ALGEBRAS

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In this paper the notion of an unbounded multiplier on a commutative Banach algebra is introduced. It is proven that, as in the case of bounded multipliers, unbounded multipliers also have Gelfand transforms. Some of the properties of these transforms are then developed. The final result of the paper is a new characterization of the bounded multipliers on A where A is a Banach algebra of the type described below.

1. **Introduction.** In general, by a multiplier on a commutative Banach algebra A one means a bounded linear operator $T: A \to A$ such that $T(xy) = xT(y)$ for all $x, y \in A$. There is an extensive literature on the subject. One can consult, for example, [2] and [3]. There does not seem to be, however, a systematic treatment of unbounded multipliers although such multipliers occur quite naturally. For example, consider the Banach algebra $L^1(-\infty, \infty)$ with the convolution product and define T by $T(f) = f'$, where the domain $\mathscr{D}(T)$ of T is the set $\{f \mid f \in L^1(-\infty, \infty)$ and f is absolutely continuous$\}$. It is easy to check that $T(fg) = fT(g)$ almost everywhere for all $f \in L^1(-\infty, \infty)$ and for all $g \in \mathscr{D}(T)$. However, T is not bounded. (Although it is closed.)

As another example, take the Banach algebra $C_0(-\infty, \infty)$ of all complex valued continuous functions on the real line which vanish at infinity with T defined by $T(f)(x) = xf(x)$. The domain $\mathscr{D}(T)$ can be taken to be the set of all functions in $C_0(-\infty, \infty)$ with compact support. Finally, let $\{T_t \mid t \geqq 0\}$ be a semi-group of class C_0 of bounded multipliers on the Banach algebra A. Then the infinitesimal generator T_0 of $\{T_t \mid t \geqq 0\}$ is an (in general) unbounded multiplier, for if $x \in \mathscr{D}(T_0)$ and $y \in A$ then

$$T_0(xy) = \lim_{h \downarrow 0} \frac{1}{h}[T(xy) - xy]$$

$$= y \lim_{h \downarrow 0} \frac{1}{h}[T(x) - x] = yT_0(x) .$$

It is the purpose of this present paper to study some of the properties of unbounded multipliers in the general setting of a commutative Banach algebra. In particular, henceforth A always denotes a regular, commutative semi-simple Banach algebra. We also assume for the rest of the paper that A has a bounded