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ON THE COMPLETENESS OF SEQUENCES OF PERTURBED POLYNOMIAL VALUES

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If S is an arbitrary sequence of positive integers, define $P(S)$ to be the set of all integers which are representable as a sum of distinct terms of S. Call a sequence S *complete* if $P(S)$ contains all sufficiently large integers, and *subcomplete* if $P(S)$ contains an infinite arithmetic progression. We will prove the following theorem: Let nth term of the integer sequence S have the form $f(n)+O(n^{\alpha})$, where f is a polynomial and where $0 \leqq \alpha < 1$; then S is subcomplete. We further show that S is complete if, in addition, for every prime p there are infinitely many terms of S not divisible by p. (We call any sequence satisfying this last property an *R-sequence*.) We will then extend these results to considerably more general sequences.

It can be shown in various ways ([3], [4]) that if f is a polynomial which maps positive integers to positive integers, then the sequence $S=\{f(1), f(2), \cdots\}$ is subcomplete, and if in addition S is an R-sequence, S is complete. In this work we use results of Folkmann's fine paper [2] to generalize these results to perturbed polynomial sequences $f(1)+t(1)$, $f(2)+t(2), \cdots$, where t is a function with sufficiently slow growth. We first state two results of [2].

THEOREM A (Folkman). *Let $A=\{a_n\}$ be a nondecreasing sequence of positive integers satisfying $a_n=O(n^{\alpha})$ for some $0 \leqq \alpha < 1$. Then A is subcomplete.*

THEOREM B (Folkman). *Let $A=\{a_n\}$ be a nondecreasing sequence of positive integers with disjoint subsequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$. Suppose that*

$$\lim_{n\to\infty} \frac{1}{b_{n+m}} \sum_{i=1}^{n} b_i = \infty \quad \text{for each} \quad m > 0 , \tag{1}$$

that $c_n > d_n$ for each n, and that the sequence $\{c_n - d_n\}$ is subcomplete. Then A is subcomplete.

We now state

THEOREM 1. *Let $S=\{s_1, s_2, \cdots\}$ be asequence of positive integers of the form $s_n=f(n)+O(n^{\alpha})$ where f is a polynomial of degree $\geqq 1$ and $0 \leqq \alpha < 1$. Then S is subcomplete.*