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THE SECOND LIE ALGEBRA COHOMOLOGY GROUP AND WEYL MODULES

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

The Lie algebra 1-cohomology of classical Lie algebras in characteristic p is nonzero at some modules. In fact, Hochschild showed that the restricted 1-cohomology is nonzero. Here I will systematically produce Weyl modules at which the (unrestricted) 2-cohomology is nonzero. This will provide examples of nonsplit abelian extensions of Lie algebras by Weyl modules in characteristic p, where the Lie algebras are the reductions modulo p of integral Chevalley forms of complex semisimple Lie algebras.

The method requires passages among cohomology groups for Lie algebras defined over the integers, rationals, and the prime field of characteristic p. In proving Theorem 1, we show that the 2-cohomology of an integral Lie algebra has nonzero p-torsion at a module whenever the reduced (modulo p) algebra has nonzero 1-cohomology at the reduced module and the extension of the integral Lie algebra to the rationals has zero for its first and second cohomology at the extension of the module by the rationals. The tensor product of the integral cohomology group with the prime field is a nonzero piece of the cohomology group of the reduced Lie algebra. This will give the examples that we seek provided that we produce modules which satisfy the conditions given above for the groups of the reduced and rational algebras. The second condition is satisfied in the classical case since the rational Lie algebra will be semisimple. As for the first condition, in [4] I gave irreducible modules in characteristic p at which the 1-cohomology is nonzero. I will show that the cohomology is also nonzero at a Weyl module associated to the irreducible module. The module that we seek for the integral Lie algebra is one that reduces modulo p to the Weyl module. I have carried out this enterprise of showing that the cohomology is nonzero at the Weyl module only for the Lie algebra of the special linear group although I believe that it can be carried out generally.

In §3, I take a particular one cocycle g (constructed in [4]) and, working with an integral cochain which reduces to g, apply the coboundary operator to obtain explicitly a p-torsion element of the integral cohomology group.

I wish to thank Professor Hochschild for his kindness and generosity over the years.