PACIFIC JOURNAL OF MATHEMATICS
Vol. 86, No. 1, 1980

LIE ALGEBRAS AND AFFINE ALGEBRAIC GROUPS

JOHN H. REINOEHL

Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

For all results obtained, attention is restricted to algebraically closed fields of characteristic zero. An affine algebraic group is said to have property (*) if the intersection of its center and its radical is unipotent. Given a Lie algebra L, a characterization is obtained of those affine algebraic groups G having property (*) for which an injection $L \to \mathscr{L}(G)$ exists whose image is algebraically dense. This is applied to obtain a result concerning the embedding of Lie algebras into algebraic Lie algebras, and to questions about the Hopf algebra of representative functions of a Lie algebra L in the case where L is algebraic.

1. Introduction. Let L be a finite-dimensional Lie algebra over a field F of characteristic zero. Let $\mathscr{U}(L)$ denote the universal enveloping algebra of L. If $\mathscr{U}(L)$ is given a topology wherein the two-sided ideals of finite codimension constitute a fundamental system of neighborhoods of 0, then the continuous dual $\mathscr{H}(L)$ of $\mathscr{U}(L)$ is the Hopf algebra of representative functions on $\mathscr{U}(L)$. $\mathscr{H}(L)$ may be viewed as a two-sided $\mathscr{U}(L)$-module as follows: for $u \in \mathscr{U}(L)$ and $f \in \mathscr{H}(L)$, $u \cdot f$ and $f \cdot u$ are defined by $(u \cdot f)(x) = f(xu)$ and $(f \cdot u)(x) = f(ux)$ for all $x \in \mathscr{U}(L)$.

An element $f \in \mathscr{H}(L)$ is termed a *semisimple element* of $\mathscr{H}(L)$ provided f is associated with a semisimple representation of L. That is the case if and only if the left $\mathscr{U}(L)$-module $\mathscr{U}(L) \cdot f$, or equivalently the right $\mathscr{U}(L)$-module $f \cdot \mathscr{U}(L)$, is semisimple. The subalgebra T of the *trigonometric elements* of $\mathscr{H}(L)$ consists of the semisimple elements of $\mathscr{H}(L)$ which are associated with representations that are trivial on the commutator ideal $[L, L]$. The following result is known from [1] and [2]. There exists a left $\mathscr{U}(L)$-stable (or equivalently, left stable under the comultiplication of $\mathscr{H}(L)$) subalgebra B of $\mathscr{H}(L)$ satisfying the following:

(1) B is finitely-generated as an F-algebra;

(2) $\mathscr{H}(L) = T \otimes B$;

(3) the subalgebra of the semisimple elements of B coincides with the portion of $\mathscr{H}(L)$ annihilated by the radical of L by left translation.

Any such subalgebra of $\mathscr{H}(L)$ is termed a *normal basic subalgebra*. Since B is finitely-generated as an F-algebra, so is the smallest Hopf