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EXTENSIONS OF PRO-AFFINE ALGEBRAIC GROUPS II

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

Introduction. We fix an algebraically closed field F of characteristic zero throughout. It is known that any pro-affine algebraic group H over F is the semidirect product $H_u \cdot H_r$ of its unipotent radical H_u and any maximal reductive subgroup H_r. This suggests, for considering extensions of a unipotent pro-affine group U over F by H, only H_u is relevant. More precisely, one is led to ask whether, given a homomorphism $H \to O(U) = \mathrm{Aut}(U)/\mathrm{Inn}(U)$ for which $\mathrm{Ext}(H, U)$ is nonempty, the restriction map $\mathrm{Ext}(H, U) \to \mathrm{Ext}(H_u, U)^H$ is bijective. The author has shown that this is the case if U is affine. We will show that for unipotent pro-affine U, the above restriction map is injective and that it is surjective in the case where $H = H_u \times H_r$, provided that $\mathrm{Ext}(H, U)$ is nonempty. We will also obtain necessary and sufficient conditions that $\mathrm{Ext}(H, U)$ be nonempty in case both H and U are affine, U unipotent.

The first two results cited above are obtained via the case where $U = A$ is abelian, unipotent and pro-affine (i.e., a pro-vector group). The main obstacle is the fact that the rational cochain groups $C^n(H_u, A)$ are not, in general, rational H_r-modules unless A is affine. This fact necessitates the technical maneuvers of the first three sections.

In §4, we give a cohomology-free proof that the restriction homomorphism $\mathrm{Ext}(H, A) \to \mathrm{Ext}(H_u, A)^H$ is an isomorphism when all groups are affine, A unipotent and abelian. We also determine when $\mathrm{Ext}(H, U)$ is nonempty, in terms of given homomorphism $H \to O(U)$, when H and U are affine, U unipotent and not necessarily abelian. The arguments of §4 where communicated to the author by Gerhard Hochschild, and I am grateful for his allowing me to include them here.

1. Some generalities on inverse limits. Throughout this section, $\mathscr{A}$ is an arbitrary but fixed directed set. All inverse systems and inverse limits have the subscript α ranging over $\mathscr{A}$. $\mathscr{C}$ is a category whose objects are at least groups and whose morphisms are group homomorphisms, in which $\{0\}$ is the zero object and exactness of a sequence has the usual meaning. The same is true of the category $\mathscr{D}$, which is large enough to contain the image of the inverse limit