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ON A REMARKABLE CLASS OF POLYHEDRA IN COMPLEX HYPERBOLIC SPACE

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

The Selberg, Piatetsky-Shapiro conjecture, now established by Margoulis, asserts that an irreducible lattice in a semi-simple group G is arithmetic if the real rank of G is greater than one. Arithmetic lattices are known to exist in the real-rank one group SO$(n, 1)$, the motion group of real hyperbolic n-space, for $n \leqq 5$. These examples due to Makarov for $n = 3$ and Vinberg for $n \leqq 5$ are defined by reflecting certain finite volume polyhedra in real hyperbolic n-space through their faces. The purpose of the present paper is to show that there are also nonarithmetic lattices in the real-rank one group PU(2, 1), the group of motions of complex hyperbolic 2-space which can be defined algebraically and leads to remarkable polyhedra. This serves to help determine the limits of the Selberg, Piatetsky-Shapiro conjecture. The analysis of these polyhedra also leads to the first known example of a compact negatively curved Riemannian space which is not diffeomorphic to a locally symmetric space.

This paper arose out of an attempt to determine the limits of validity of the Selberg, Piatetsky-Shapiro conjecture on the arithmeticity of lattice subgroups. In 1960 A. Selberg conjectured that apart from some exceptional G, an irreducible noncocompact lattice subgroup Γ of a semi-simple group G is arithmetic ("irreducible" in the sense that Γ is not commensurable with a direct product of its intersections with factors of G). Later Piatetsky-Shapiro conjectured: An irreducible lattice of a semi-simple group G is arithmetic if $\boldsymbol{R}$-rank $G > 1$.

The Selberg, Piatetsky-Shapiro conjecture was settled affirmatively by G. A. Margoulis in the striking paper that he submitted to the 1974 International Congress of Mathematicians in Vancouver.

The simple groups of $\boldsymbol{R}$-rank 1 are (up to a local isomorphism)

$$\mathrm{SO}(n, 1), \mathrm{SU}(n, 1), \mathrm{SP}(n, 1), F_4$$

which act as isometries on the hyperbolic space

$$\boldsymbol{R}h^n, \boldsymbol{C}h^n, \boldsymbol{H}h^n, \boldsymbol{O}h^2$$

over the real numbers $\boldsymbol{R}$, the complex numbers $\boldsymbol{C}$, the quaternions (or Hamiltonians) $\boldsymbol{H}$, the octonians (or Cayley numbers) $\boldsymbol{O}$ respectively. Nonarithmetic lattices in SO(2, 1)$(=\mathrm{SL}_2(\boldsymbol{R})/\pm 1)$ have been